Exploration 2.3.1 Extension: The Mechanics of Conics

Circle

1. The equation of a circle in standard form is $(x-h)^2+(y-k)^2=r^2$. Write the equation of a circle, in standard form, with a center at $(-5,3)$ and radius 4 units.

2. The general form of a conic section is $Ax^2+Cy^2+Dx+Ey+F=0$. Using the equation from part 1, convert the standard form to general form. Hint: expand the binomials.

3. Convert the following equation of a circle in general form to standard form and identify the center and radius. Hint: start by completing the square for both variables.
   $x^2+y^2-4x+2y-20=0$

Ellipse

4. The standard form of an ellipse is $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$. Compare this to the standard form of the circle. What do you notice?

5. The following equation represents an ellipse. Put the equation in to standard form. Hint: ensure your equation is set equal to 1 using division.

   $x^2 + 9y^2 - 2x + 36y + 28 = 0$

6. Based on the equation you just put in to standard form, make a conjecture as to where the center is located and how the a^2 and b^2 values affect the graph of the ellipse.

7. Using a graphing utility, like Desmos or GeoGebra, graph the equation of the ellipse and confirm your conjectures. If necessary, make a new conjecture based on your findings after graphing.

8. The major axis of an ellipse connects the two vertices and the minor axis connects the two co-vertices. Write a statement(s) relating the distances of the major and minor axes to the *a*- and *b*-values. List the coordinates of the vertices and the co-vertices.

9. In Exploration 2.3.1, you verified the following for the ellipse: $c^2 = a^2 - b^2$. The *c*-value determines the distance from the center of an ellipse to its foci (fixed points), which are located on the major axis. List the coordinates of the foci.

Hyperbola

10. The following equation represents a hyperbola. Put the equation in to standard form.

    (Standard form of a hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$)

    $25x^2-16y^2+100x-96y=444$

11. Based on the equation you just put in to standard form, make a conjecture as to where the center is located and how the a^2 and b^2 values affect the graph of the hyperbola. What happens to the hyperbola if the x^2-term is negative?

12. Using a graphing utility, like Desmos or GeoGebra, graph the equation of the hyperbola and confirm your conjectures. If necessary, make a new conjecture based on your findings after graphing.

13. The transverse axis of a hyperbola connects the two vertices. List the coordinates of the vertices.

14. In Exploration 2.3.1, you verified the following for the hyperbola: $c^2 = a^2 + b^2$. The *c*-value determines the distance from the center of a hyperbola to its foci (fixed points), which are located on the transverse axis. List the coordinates of the foci.

Parabola

15. The following equation represents a parabola. Put the equation in to standard form. (Standard form of a parabola: $(x-h)^2 = 4p(y-k)$ or $(x-h)^2 = 4c(y-k)$) Note: various texts interchange *p* and *c*.

$y = x^2 - 6x + 13$

16. Based on the equation you just put in to standard form, make a conjecture as to where the vertex is located, the directrix is located, and how the $4p$ value affects the graph of the parabola. What happens to the parabola if the x^2-term is negative?

17. Using a graphing utility, like Desmos or GeoGebra, graph the equation of the parabola and confirm your conjectures. If necessary, make a new conjecture based on your findings after graphing.