HW 1.2.1: Function Foundations (Domain and Range)

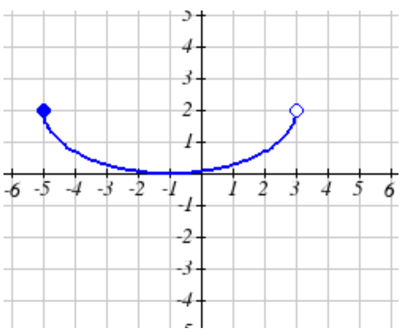
Inequality	Set Builder Notation	Interval notation
$5 < h \leq 10$	$\{h \mid 5 < h \leq 10\}$	(5, 10]
$5 \leq h < 10$	$\{h \mid 5 \leq h < 10\}$	[5, 10)
$5 < h < 10$	$\{h \mid 5 < h < 10\}$	(5, 10)
$h < 10$	$\{h \mid h < 10\}$	$(-\infty, 10)$
$h \geq 10$	$\{h \mid h \geq 10\}$	$[10, \infty)$
all real numbers	$\{h \mid h \in \mathbb{R}\}$	$(-\infty, \infty)$

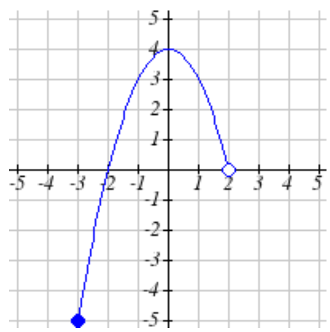
As an inequality it is: $1 \leq x \leq 3$ or $x > 5$

In set builder notation: $\{x \mid 1 \leq x \leq 3 \text{ or } x > 5\}$

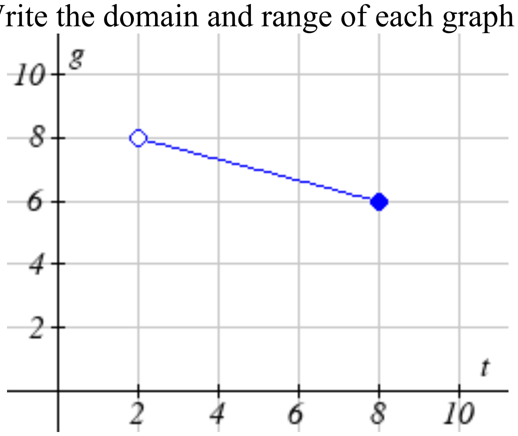
In interval notation: $[1,3] \cup (5, \infty)$

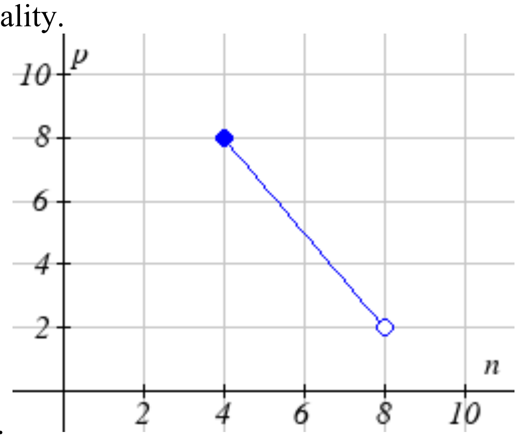
Write the domain and range of the function using interval notation.

1. 

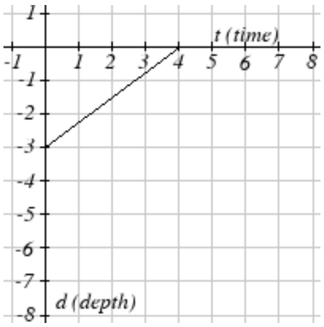
2. 

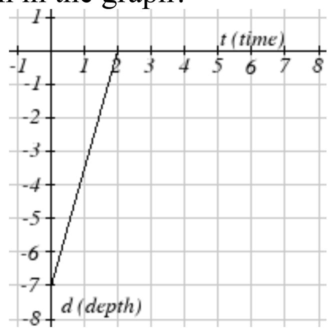
Write the domain and range of each graph as an inequality.

3. 

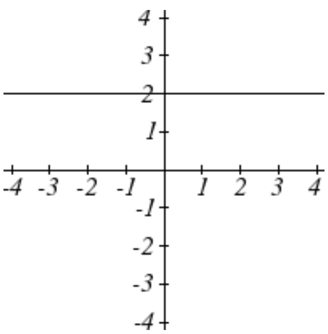
4. 

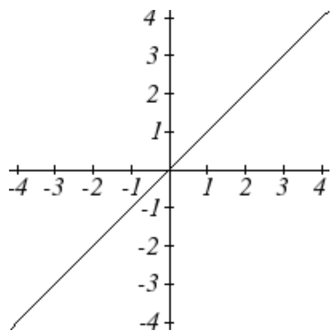
Suppose that you are holding your rubber ducky under the water. You release it and it begins to ascend. The graph models the depth of the ducky as a function of time, stopping once the duck surfaces. What is the domain and range of the function in the graph?

5. 

6. 

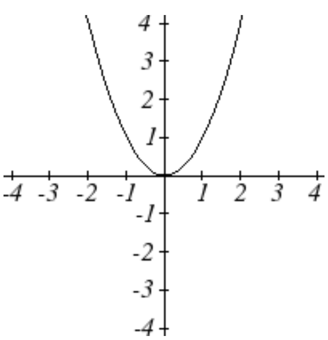
List the Domains and Ranges of several Parent functions

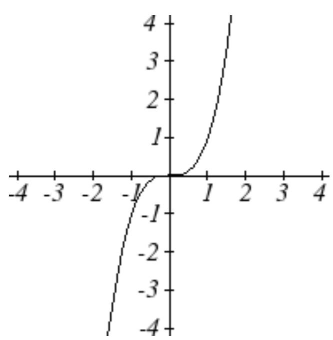




Constant Function: $f(x) = c$
Domain:
Range:

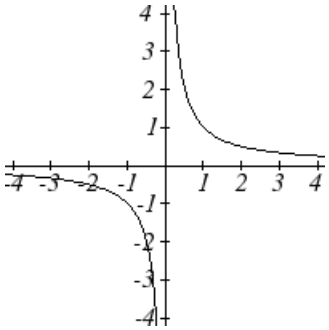
Identity Function: $f(x) = x$
Domain:
Range:





Quadratic Function: $f(x) = x^2$
Domain:
Range:

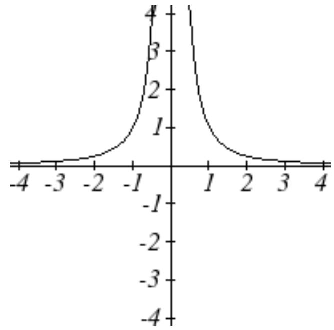
Cubic Function: $f(x) = x^3$
Domain:
Range:



Reciprocal: $f(x)=\dfrac{1}{x}$

Domain:

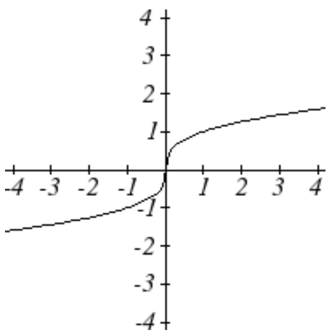
Range:



Reciprocal squared: $f(x)=\dfrac{1}{x^2}$

Domain:

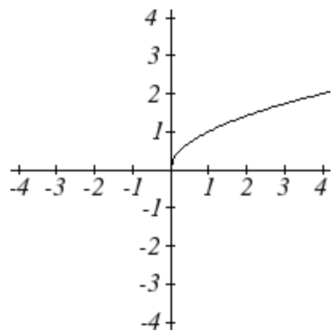
Range:



Cube Root: $f(x)=\sqrt[3]{x}$

Domain:

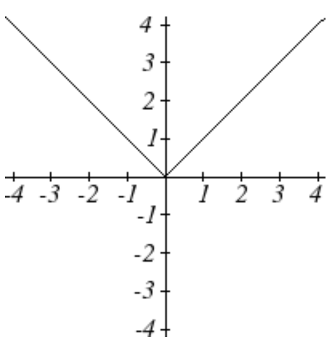
Range:



Square Root: $f(x)=\sqrt{x}$

Domain:

Range:



Absolute Value Function: $f(x)=|x|$

Domain:

Range:

Find the domain of each function

7. $f\left(x\right)=2\sqrt{x+1}$

8. $f\left(x\right)=4\sqrt{x-4}$

9. $f\left(x\right)=7-\sqrt{9-3x}$

10. $f\left(x\right)=2+\sqrt{15-5x}$

11. $f\left(x\right)=\dfrac{4}{x+2}$

12. $f\left(x\right)=\dfrac{5}{x-10}$

13. $f\left(x\right)=\dfrac{5x-2}{2x+1}$

14. $f\left(x\right)=\dfrac{4x+4}{x+3}$

15. $f\left(x\right)=\dfrac{\sqrt{x+9}}{x-9}$

16. $f\left(x\right)=\dfrac{\sqrt{x-1}}{x-3}$

17. $f\left(x\right)=\dfrac{x+1}{x^2+11x-12}$

18. $f\left(x\right)=\dfrac{x-2}{x^2+2x-15}$

Selected Answers

1. The domain is $[-5, 3)$; the range is $[0, 2]$

3. The domain is $2 < x \leq 8$; the range is $6 \leq y < 8$

5. The domain is $0 \leq x \leq 4$; the range is $-3 \leq y \leq 0$

Constant Function: $f(x) = c$
The domain here is not restricted; x can be anything. When this is the case we say the domain is all real numbers. The outputs are limited to the constant value of the function.
Domain: $(-\infty, \infty)$
Range: $[c]$
Since there is only one output value, we list it by itself in square brackets.

Identity Function: $f(x) = x$
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

Quadratic Function: $f(x) = x^2$
Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Multiplying a negative or positive number by itself can only yield a positive output.

Cubic Function: $f(x) = x^3$
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

Reciprocal: $f(x) = \frac{1}{x}$
Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(-\infty, 0) \cup (0, \infty)$
We cannot divide by 0 so we must exclude 0 from the domain.
One divide by any value can never be 0, so the range will not include 0.

Reciprocal squared: $f(x) = \frac{1}{x^2}$
Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(0, \infty)$
We cannot divide by 0 so we must exclude 0 from the domain.

Cube Root: $f(x) = \sqrt[3]{x}$
Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

<u>Square Root</u>: $f(x) = \sqrt[2]{x}$, commonly just written as, $f(x) = \sqrt{x}$
Domain: $[0, \infty)$
Range: $[0, \infty)$
When dealing with the set of real numbers we cannot take the square root of a negative number so the domain is limited to 0 or greater.

<u>Absolute Value Function</u>: $f(x) = |x|$
Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Since absolute value is defined as a distance from 0, the output can only be greater than or equal to 0.

7. Since the function is not defined when there is a negative number under the square root, x cannot be less than -1 (it can be equal to -1, because $\sqrt{0}$ is defined). So the domain is $x \geq -1$. Because the inputs are limited to all numbers greater than -1, the number under the square root will always be positive, so the outputs will be limited to positive numbers. So the range is $f(x) \geq 0$.

9. Since the function is not defined when there is a negative number under the square root, x cannot be greater than 3 (it can be equal to 3, because $\sqrt{0}$ is defined). So the domain is $x \leq 3$. Because the inputs are limited to all numbers less than 3, the number under the square root will always be positive, and there is no way for 7 minus a positive number to equal more than seven, so the outputs can be any number less than 7. So the range is $f(x) \leq 7$.

11. Since the function is not defined when there is division by zero, x cannot equal -2. So the domain is all real numbers except -2, or $\{x | x \in \mathbb{R}, x \neq -2\}$.

13. Since the function is not defined when there is division by zero, x cannot equal $-1/2$. So the domain is all real numbers except $-1/2$, or $\{x | x \in \mathbb{R}, x \neq -1/2\}$.

15. Since the function is not defined when there is a negative number under the square root, x cannot be less than -9 (it can be equal to -9, because $\sqrt{0}$ is defined). Since the function is also not defined when there is division by zero, x also cannot equal 9. So the domain is all real numbers greater than -9 excluding 9, or $\{x | x \geq -9, x \neq 9\}$.

17. It is easier to see where this function is undefined after factoring the denominator. This gives $f(x) = \frac{x+1}{(x-1)(x+12)}$. It then becomes clear that the denominator is undefined when $x = -12$ and when $x = 1$ because they cause division by zero. Therefore, the domain is $\{x | x \in \mathbb{R}, x \neq -12, x \neq 1\}$.