HW 1.5.3: Piecewise Functions

Given each function, evaluate: $f(-1),\ f(0),\ f(2),\ f(4)$

1. $f(x)=\begin{cases} 4x+5 & \text{if } x<0 \\ 4x+8 & \text{if } x\geq 0 \end{cases}$

2. $f(x)=\begin{cases} 6x-12 & \text{if } x<0 \\ 6x-16 & \text{if } x\geq 0 \end{cases}$

3. $f(x)=\begin{cases} x^2-5 & \text{if } x<4 \\ 8+|x-9| & \text{if } x\geq 4 \end{cases}$

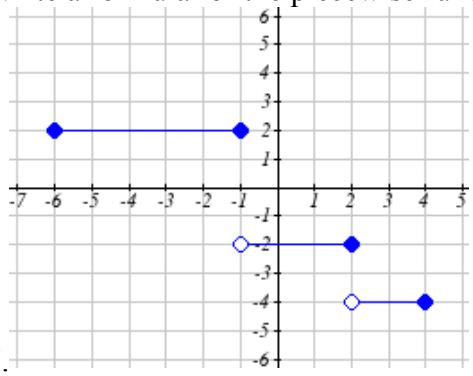
4. $f(x)=\begin{cases} 2-x^3 & \text{if } x<-2 \\ \sqrt{x+3} & \text{if } x\geq -2 \end{cases}$

5. $f(x)=\begin{cases} 9x & \text{if } x<0 \\ 14 & \text{if } 0\leq x\leq 1 \\ 2x^2 & \text{if } x>1 \end{cases}$

6. $f(x)=\begin{cases} 3x^3+8 & \text{if } x<0 \\ 10 & \text{if } 0\leq x\leq 1 \\ 7x+14 & \text{if } x>1 \end{cases}$

Write a formula for the piecewise function graphed below.

7.



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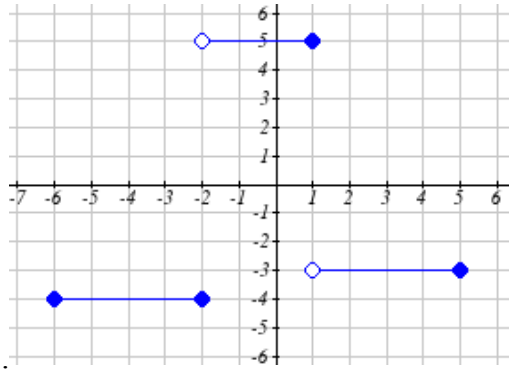
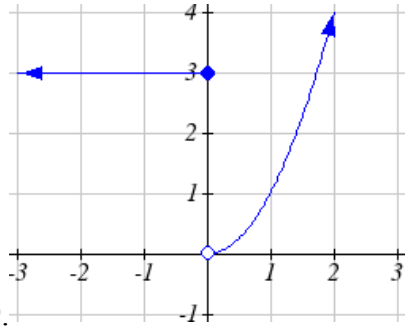
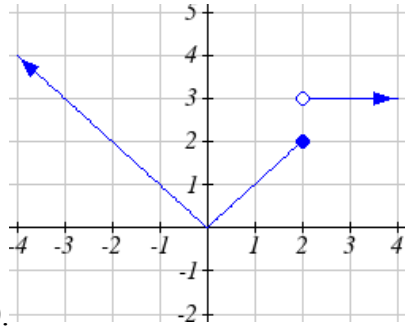
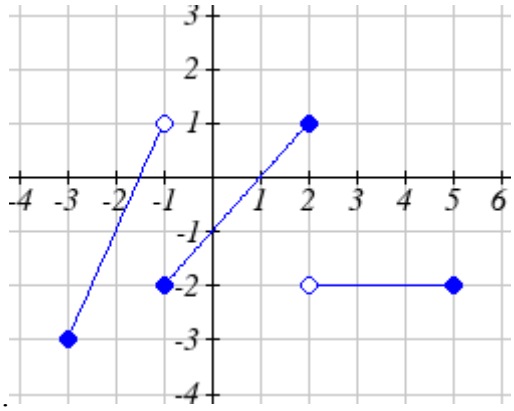
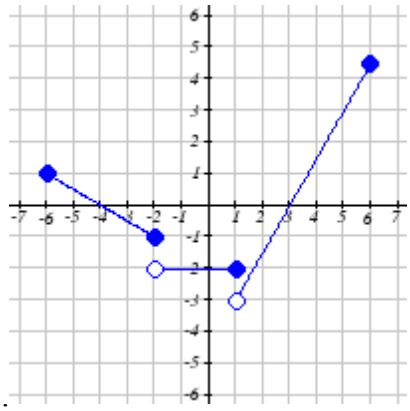


9. 

10. 

11. 

12. 

Sketch a graph of each piecewise function

13. $f(x)=\begin{cases} |x| & \text{if } x<2 \\ 5 & \text{if } x\geq 2 \end{cases}$

14. $f(x)=\begin{cases} 4 & \text{if } x<0 \\ \sqrt{x} & \text{if } x\geq 0 \end{cases}$

15. $f(x)=\begin{cases} x^2 & \text{if } x<0 \\ x+2 & \text{if } x\geq 0 \end{cases}$

16. $f(x)=\begin{cases} x+1 & \text{if } x<1 \\ x^3 & \text{if } x\geq 1 \end{cases}$

17. $f(x)=\begin{cases} 3 & \text{if} \quad x\le -2 \\ -x+1 & \text{if} \quad -2<x\le 1 \\ 3 & \text{if} \quad x>1 \end{cases}$

18. $f(x)=\begin{cases} -3 & \text{if} \quad x\le -2 \\ x-1 & \text{if} \quad -2<x\le 2 \\ 0 & \text{if} \quad x>2 \end{cases}$

19. For n copies of the book *A Day in the Life*, a print on-demand company charges $C(n)$ dollars, where $C(n)$ is determined by the formula

$$C(n)=\begin{cases} 20n & \text{if} \quad 1\le n\le 30 \\ 15.50n & \text{if} \quad 30<n\le 45 \\ 10n & \text{if} \quad n>45 \end{cases}$$

a. Find and interpret $C(15)$

b. How much does it cost to order 45 copies of the book? What about 46 copies?

c. Your answer to 19b should get you thinking. Suppose a bookstore estimates it will sell 45 copies of the book. How many books can, in fact, be ordered for the same price as those 45 copies? (Round your answer to a whole number of books.)

20. An on-line clothing retailer charges shipping costs according to the following formula

$$S(n)=\begin{cases} 2n+5.5 & \text{if} \quad 1\le n<25 \\ 0 & \text{if} \quad n\ge 25 \end{cases}$$

where n is the number of clothing items and $S(n)$ is the shipping cost in dollars.

a. What is the cost to ship 15 clothing items?

b. What is the significance of the formula S(n) = 0 for n ≥ 25?

21. The cost C (in dollars) to send m text messages a month on a mobile phone plan is modeled by

$$C(m)=\begin{cases}30 & \text{if } \ 0\le m\le 2500\\ 30+0.2(m-2500) & \text{if } \ m>2500\end{cases}$$

a. How much does it cost to send 1000 text messages per month with this plan?

b. How much does it cost to send 90 text messages each day for a month with this plan?

c. Explain the terms of the plan verbally.

22. The set of integers as $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$. The **greatest integer of *x***, denoted by $\lfloor x \rfloor$, is defined to be the largest integer k with $k \le x$.

a. Find $\lfloor 0.625 \rfloor$, $\lfloor 141 \rfloor$, $\lfloor -4.002 \rfloor$, and $\lfloor \pi + 2 \rfloor$

b. Discuss with your classmates how $\lfloor x \rfloor$ may be described as a piecewise defined function. **HINT:** There are infinitely many pieces!

c. Is $\lfloor a \rfloor + \lfloor b \rfloor = \lfloor a + b \rfloor$ always true? What if a or b is an integer? Test some values, make a conjecture, and explain your result.

Selected Answers:

1. $f(-1) = 1; f(0) = 8; f(2) = 16; f(4) = 24$

3. $f(-1) = -4; f(0) = -5; f(2) = -1; f(4) = 13$

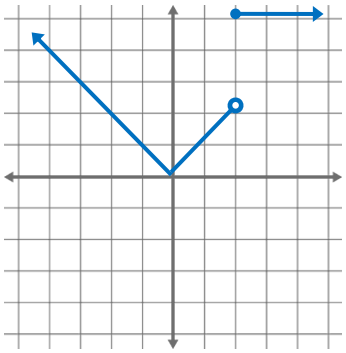
5. $f(-1) = -9; f(0) = 14; f(2) = 8; f(4) = 32$

7. $f(x) = \begin{cases} 2 & if \quad -6 \le x \le -1 \\ -2 & if \quad -1 < x \le 2 \\ -4 & if \quad 2 < x \le 4 \end{cases}$

9. $f(x) = \begin{cases} 3 & if \quad x \le 0 \\ x^2 & if \quad x > 0 \end{cases}$

11. $f(x) = \begin{cases} 2x + 3 & if \quad 3 \le x < -1 \\ x - 1 & if \quad -1 \le x \le 2 \\ -3 & if \quad 2 < x \le 5 \end{cases}$

13.



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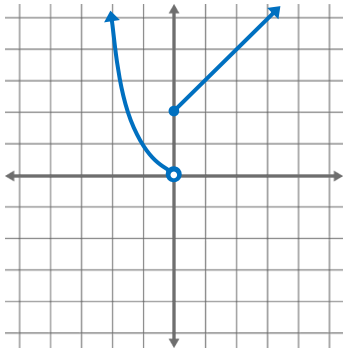

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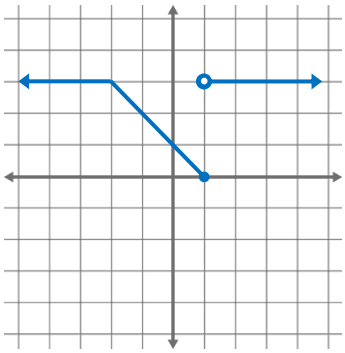

20. a. $S(10) = 35.5$, so it costs \$35.50 to ship 15 clothing items.

b. There is free shipping on orders of 25 or more clothing items.

22.a. $\lfloor 0.625 \rfloor = 0$, $\lfloor 141 \rfloor = 141$, $\lfloor -4.002 \rfloor = -5$, and $\lfloor \pi + 2 \rfloor = 5$