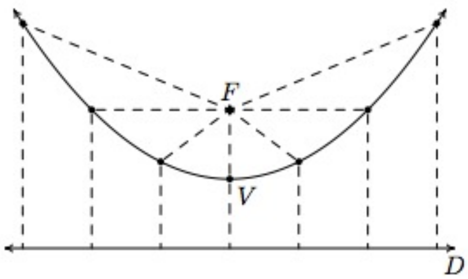
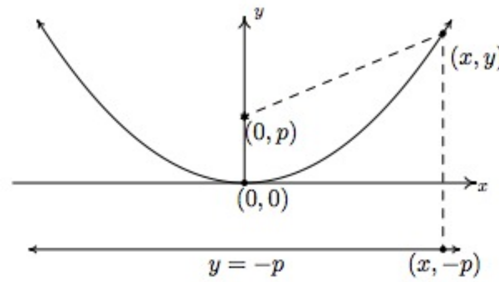
2.3 Conic Sections – Parabola

<u>Parabola</u> (locus definition) Set of all points equidistant from a Focus to a Directrix.





<u>Standard Form of a Parabola:</u>

Vertical Parabola

$(x-h)^2 = 4p(y-k)$

Horizontal Parabola

$(y-k)^2 = 4p(x-h)$

vertex = (h, k)

p = distance from vertex to focus

Vertical Parabola:

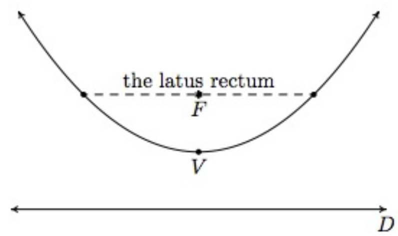
Focus: $(h, k+p)$

Directrix: $y = k-p$

Horizontal Parabola:

Focus: $(h+p, k)$

Directrix: $x = h-p$



4p = Latus Rectum = focal diameter of the parabola

Ex. Graph $(x+1)^2 = -8(y-3)$

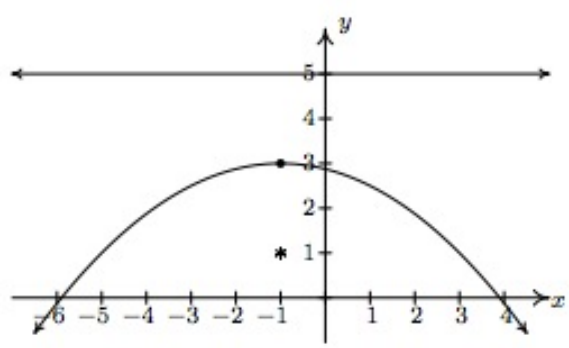
Vertical Parabola

Vertex: $(-1,3)$

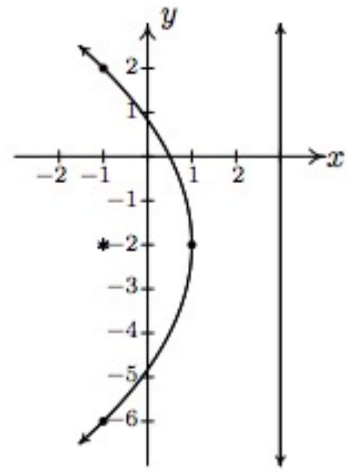
p = -2

Focus: $(-1,1)$

Directrix: $y=5$



Ex. Consider the equation $y^2+4y+8x=4$. Put this equation into standard form and identify the vertex, focus, directrix, and graph.



$$y^2+4y+8x=4$$

$$y^2+4y=-8x+4$$

complete the square in y only

$$y^2+4y+4=-8x+4+4$$

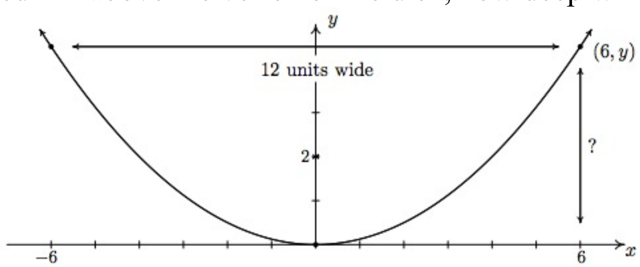
factor

$$(y+2)^2=-8x+8$$

$$(y+2)^2=-8(x-1)$$

Vertex: $(1,-2)$
Focus: $(-1,-2)$
Directrix: $x=3$

Ex. A satellite dish is to be constructed in the shape of a paraboloid of revolution. If the receiver placed at the focus is located 2 ft above the vertex of the dish, how deep will the dish be?



Use (0, 0) as the vertex, (6, y) a point on the parabola, p = 2, and plug into the standard form of a vertical parabola.

$$(x-0)^2=4p(y-0)$$

$$x^2=4(2)y$$

$$(6)^2=8y$$

$$\frac{36}{8}=y$$

$$\frac{9}{2}=y=4.5$$

In Exercises 1-6, sketch the graph of the given parabola. Find the vertex, focus, and directrix. Include the endpoints of the latus rectum in your sketch.

1. $(x-3)^2=-16y$

2. $\left(x+\frac{8}{5}\right)^2=4\left(y+\frac{9}{4}\right)$

3. $(y-8)^2=-10(x+7)$

4. $(y+4)^2=4x$

5. $(x-4)^2=2(y+6)$

6. $(y-1)^2=24(x-3)$

In Exercises 7-10, put the equation into standard form and identify the vertex, focus, and directrix.

7. $y^2-6y-36x+117=0$

8. $2x^2+4x+3y-4=0$

9. $x^2+6x-9y+81=0$

10. $x^2-8x+6y+4=0$

In Exercises 11-12, find an equation for the parabola which fits the given criteria.

11. Vertex (3,0), focus (0,0)

12. Focus (12,3), directrix x=6

13. The mirror in Alan's flashlight is a paraboloid of revolution. If the mirror is 8 centimeters in diameter and 4.5 centimeters deep, where should the light bulb be placed so it is at the focus of the mirror?

14. A parabolic TV antenna is constructed by taking a flat sheet of metal and bending it into a parabolic shape. If the cross of the antenna is a parabola which is 50 centimeters wide and 30 centimeters deep, where should the receiver be placed to maximize reception?

15. A parabolic arch is constructed which is 8 feet wide at the base and 13 feet tall in the middle. Find the height of the arch exactly 2 feet in from the base of the arch.