HW 3.1.2: Compound Interest

n Compounds per year

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

Compound Continuously

$$A = Pe^{rt}$$

A= Amount
P = Principal (initial investment)
r = APR (annual percent rate, decimal)
t = number of years
n = number of compounds per year

Do each of the bullets for Exercises 1-6.

- Find the amount, $A(t)$, in the account as a function of the term of the investment t in years.
- Determine how much is in the account after 5 years, 10 years, and 30 years. Round your answers to the nearest cent.
- Determine how long will it take for the initial investment to double. Round your answer to the nearest year.

1. $500 is invested in an account, which offers 0.75%, compounded monthly.

2. $500 is invested in an account, which offers 0.75%, compounded continuously.

3. $1000 is invested in an account, which offers 1.25%, compounded quarterly.

4. $1000 is invested in an account, which offers 1.25%, compounded continuously.

5. $5000 is invested in an account, which offers 2.125%, compounded daily.

6. $5000 is invested in an account, which offers 2.125%, compounded continuously.

7. Look back at your answers to Exercises 1-6. What can be said about the differences between monthly, daily, or quarterly compounding and continuously compounding the interest in those situations?

8. How much money needs to be invested now to obtain \$2000 in 3 years if the interest rate in a savings account is 0.25%, compounded continuously? Round your answer to the nearest cent.

9. How much money needs to be invested now to obtain \$5000 in 10 years if the interest rate in a CD is 2.25%, compounded monthly? Round your answer to the nearest cent.

10. On May 31, 2014, the Annual Percentage Rate listed at Jeff's bank for regular savings accounts was 0.25%, compounded monthly.
    a. If $P = 2000$ what is $A(8)$?

    b. Solve the equation $A(t) = 4000$ for t.

    c. What principal P should be invested so that the account balance is \$2000 in three years?

11. Show that the time it takes for an investment to double in value does not depend on the principal P, but rather, depends only on the APR and the number of compoundings per year. Let $n = 12$ and with the help of your classmates compute the doubling time for a variety of rates r. Then look up the Rule of 72 and compare your answers to what the rule says. If you are really interested in Financial Mathematics, you could also compare and contrast the Rule of 72 with the Rule of 70 and the Rule of 69.

Answers:

1.
- $A(t)=500\left(1+\frac{0.0075}{12}\right)^{12t}$
- $A(5)\approx \$519.10,\quad A(10)\approx \$538.93,\quad A(30)\approx \$626.12$
- $92.45\approx 92$ years for the investment to double.

2.
- $A(t)=500e^{0.0075t}$
- $A(5)\approx \$519.11,\quad A(10)\approx \$538.94,\quad A(30)\approx \$626.16$
- $92.42\approx 92$ years for the investment to double.

3.
- $A(t)=1000\left(1+\frac{0.0125}{4}\right)^{4t}$
- $A(5)\approx \$1064.39,\quad A(10)\approx \$1132.93,\quad A(30)\approx \$1454.14$
- $55.54\approx 56$ years for the investment to double.

4.
- $A(t)=1000e^{0.0125t}$
- $A(5)\approx \$1064.49,\quad A(10)\approx \$1133.15,\quad A(30)\approx \$1454.99$
- $55.45\approx 55$ years for the investment to double.

5.
- $A(t)=5000\left(1+\frac{0.02125}{365}\right)^{365t}$
- $A(5)\approx \$5560.48,\quad A(10)\approx \$6183.79,\quad A(30)\approx \$9458.55$
- $32.62\approx 33$ years for the investment to double.

6.
- $A(t)=5000e^{0.02125t}$
- $A(5)\approx \$5560.50,\quad A(10)\approx \$6183.83,\quad A(30)\approx \$9458.73$
- $32.62\approx 33$ years for the investment to double.

8. $1985.06
9. $3993.42
10. a. $2040.40 b. 277.29 years c. $1985.06