HW 3.2.1: Half-Life

$$N(t) = N_0 e^{-\frac{t \ln 2}{h}} \qquad \text{or} \qquad N(t) = N_0 \left(\frac{1}{2}\right)^{\frac{t}{h}}$$

N_0 = initial amount
t = time in (unit given as half-life)
h = length of half-life

In Exercises 1-5, complete the following:

- Create a function, $N(t)$, for the amount of isotope after t years.
- Determine how long it takes for 80% of the material to decay. Round answers to two decimal places. (Make sure you are comfortable using both half-life formulas.)

1. Cobalt 60, used in food irradiation, initial amount 60 grams, half-life of 5.27 years.

2. Fluorine 18, used in medical radiotracer, initial amount 5 milligrams, half-life 110 minutes.

3. Iodine 131, used in nuclear medicine, initial amount 85 milligrams, half-life 8 days.

4. Americium 241, used in smoke detectors, initial amount 0.34 micrograms, half-life 432.7 years.

5. Carbon 14, used for radiocarbon dating, initial amount 15 kg grams, half-life 5,700 years.

6. How many half-lives have passed if 6.25% of the material remains? (Do not use a calculator.)

Answers:

1. $N(t)=60e^{-\frac{t\ln 2}{5.27}}$ or $N(t)=60\left(\frac{1}{2}\right)^{\frac{t}{5.27}}$; 12.237 years

2. $N(t)=5e^{-\frac{t\ln 2}{110}}$ or $N(t)=5\left(\frac{1}{2}\right)^{\frac{t}{110}}$; 255.402 minutes

3. $N(t)=85e^{-\frac{t\ln 2}{8}}$ or $N(t)=85\left(\frac{1}{2}\right)^{\frac{t}{8}}$; 18.575 days

4. $N(t)=0.34e^{-\frac{t\ln 2}{432.7}}$ or $N(t)=0.34\left(\frac{1}{2}\right)^{\frac{t}{432.7}}$; 1004.7 years

5. $N(t)=15e^{-\frac{t\ln 2}{5700}}$ or $N(t)=15\left(\frac{1}{2}\right)^{\frac{t}{5700}}$; 13,235 years

6. 50% decay = 1 half-life, $\left(\frac{1}{2}\right)^2=\frac{1}{4}=25\%$ remaining = 2 half-lives, $\left(\frac{1}{2}\right)^3=\frac{1}{8}=12.5\%$ remaining = 3 half-lives, $\left(\frac{1}{2}\right)^4=\frac{1}{16}=6.25\%$ remaining = 4 half-lives