# HW 3.3.2: The Logistic Growth Model

A fundamental population growth model in ecology is the *logistic model.* In one respect, logistic population growth is more realistic than exponential growth because logistic growth is bounded. We can write the logistic model as,

$$P(t)=\frac{P_0\cdot K}{P_0+\left(K-P_0\right)\cdot e^{-rt}} \quad \text{or} \quad P(t)=\frac{K}{1+\dfrac{K-P_0}{P_0}e^{-rt}},$$

Where $P(t)$ is the population size at time $t$, $P_0$ is the initial population size, $K$ is the carrying capacity of the environment, defined as the maximum population size an environment can support, and $r$ is a constant representing the rate of population growth or decay.

For all exercises, when solving for a variable, give both the exact (without calculator) and approximate (with calculator) answers.

1. The population of Sasquatch in Bigfoot County is modeled by $P(t)=\dfrac{120}{1+4e^{-0.05t}}$, where $P(t)$ is the population of Sasquatch $t$ years after 2010.

(a) Find and interpret $P(0)$.

(b) Find the population of Sasquatch in Bigfoot County in 2013. Round your answer to the nearest Sasquatch.

(c) When will the population of Sasquatch in Bigfoot County reach 60? Round your answer to the nearest year.

(d) Find and interpret the end behavior of the graph of $y=P(t)$. Check your answer using a graphing utility.

2. A conservation organization releases 100 animals of an endangered species into a game preserve. The organization believes that the preserve has a carrying capacity of 1000 animals and that the growth of the pack will be modeled by the logistic curve:

$$P(t)=\frac{1000}{1+9e^{-0.1656t}}$$

where $t$ is measured in months.

(a) Estimate the population after 5 months.

(b) After how many months will the population be 500?

3. On a college campus of 5000 students, one student returns from vacation with a contagious and long-lasting flu virus. The spread of the virus is modeled by

$$F(t)=\frac{5000}{1+4999e^{-0.8t}}$$

where $F(t)$ is the total number of students infected after $t$ days. The college will cancel classes if 40% or more of the students are infected.

(a) Graph the function and sketch the curve, label an appropriate domain and range.

(b) How many students are infected after 5 days?

(c) After how many days will the college cancel classes?

4. Of a group of 200 college men the number $N$ who are taller than $x$ inches is given below:

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x$: | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 (6 ft) | 73 | 74 | 75 | 76 |
| $N$: | 197 | 195 | 184 | 167 | 137 | 101 | 71 | 43 | 25 | 11 | 4 | 2 |

(a) Construct a scatter plot for this data and explain why a logistic model is appropriate for this data.

If you did logistic regression you would see $N(x)=\dfrac{207}{1+\left(1.285\times10^{-21}\right)e^{0.686x}}$

(b) How is the shape of this curve different from that in number 3(a)?

(c) How many men in the group are over 69.5 inches tall?

(d) The tallest 85 men are all grouped together, all of these men are at least _____ inches tall. Round to the nearest tenth of an inch.

Selected answers:

1.

(a) $P(0) = 24$; There are 24 Sasquatch in Bigfoot County in 2010.

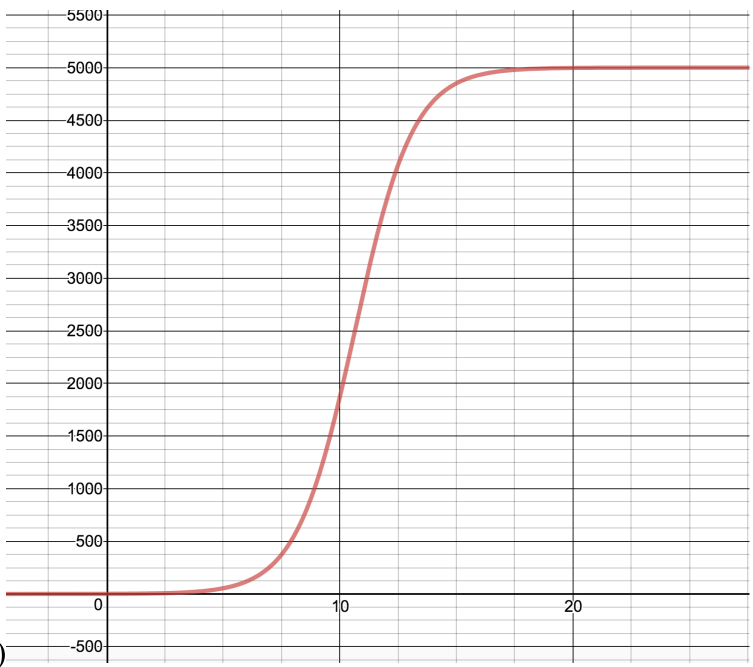
(b) $P(3) = \dfrac{120}{1+4e^{-0.15}}$; $P(3) \approx 27$ Sasquatch in 2013

(c) $t = \dfrac{\ln 4}{0.05}$; The Sasquatch population will reach 60 in the year 2038

(d) As $t \to \infty, P(t) = 120$. As time goes by, the Sasquatch Population in Bigfoot County will approach 120. Graphically, $y = P(x)$ has a horizontal asymptote y = 120.

3.

(a)




Domain: $(-\infty, \infty)$ Range: $(0, 5000)$

(b) $F(5) = \dfrac{5000}{1+4999e^{-4}}$; 54 students will be infected after five days.

(c) $t = \dfrac{\ln\left(\dfrac{9998}{3}\right)}{0.8}$ ;10.14 days