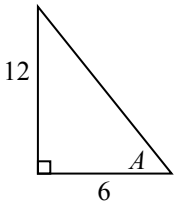
HW 4.0: Simple Triangle Trigonometry

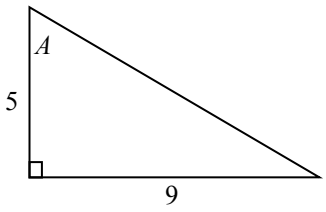
Note: pictures may not be drawn to scale.

For each of the triangles below, find $\sin(A)$, $\cos(A)$, $\tan(A)$, $\sec(A)$, $\csc(A)$, and $\cot(A)$.

1.



2.



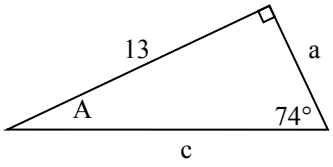
Given the trigonometric equation, find $\sin(A)$, $\cos(A)$, $\tan(A)$, $\sec(A)$, $\csc(A)$, and $\cot(A)$. (Hint: draw a triangle.)

3. $\cos A = \frac{24}{25}$

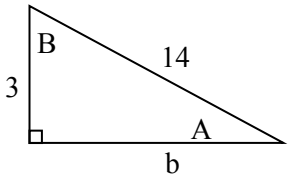
4. $\sin A = \frac{\sqrt{12}}{4}$

In each of the following triangles, solve for the unknown sides and angles.

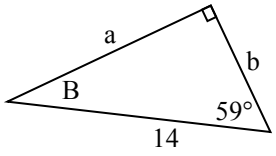
5.



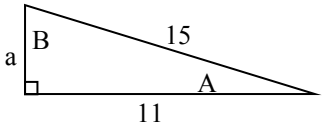
6.



7.



8.



9. A 25-ft ladder leans against a building so that the angle between the ground and the ladder is 70°. How high does the ladder reach up the side of the building?

10. A 50-ft ladder leans against a building so that the base of the ladder is 15 feet from the building. What angle does the ladder make with the building?

11. The angle of elevation to the top of a building in New York is found to be 5 degrees from the ground at a distance of 2 miles from the base of the building. Using this information, find the height of the building.

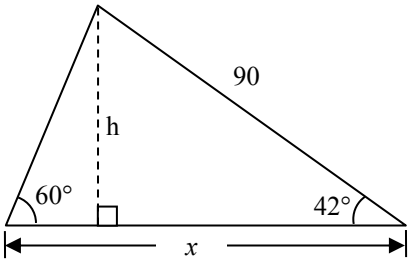
12. The angle of elevation to the top of a building in Seattle is found to be θ degrees from the ground at a distance of 3 miles from the base of the building. The height of this building is 820 feet. What is the angle of elevation?

13. A radio tower is located 600 feet from a building. From a window in the building, a person determines that the angle of depression to the bottom of the tower is 40° and that the angle of elevation to the top of the tower is 19°. How tall is the tower?

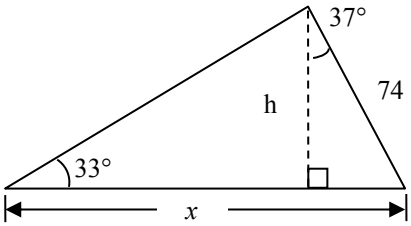
14. A 70 foot tall monument is located in the distance. From a window in a building, a person determines that the angle of elevation to the top of the monument is 13° and that the angle of depression to the bottom of the monument is 8°. How far is the person from the monument?

15. There is an antenna on the top of a building. From a location 500 feet from the base of the building, the angle of elevation to the top of the building is measured to be 40°. From the same location, the angle of elevation to the top of the antenna is measured to be 43°. Find the height of the antenna.

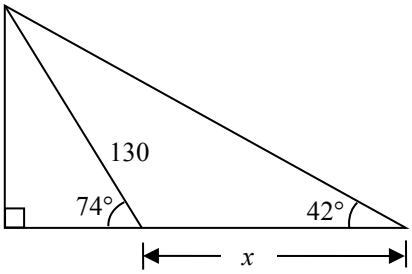
16. Find the length x.



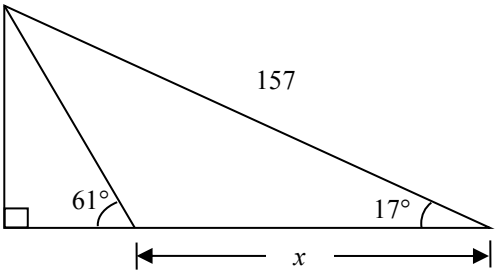
17. Find the length x.



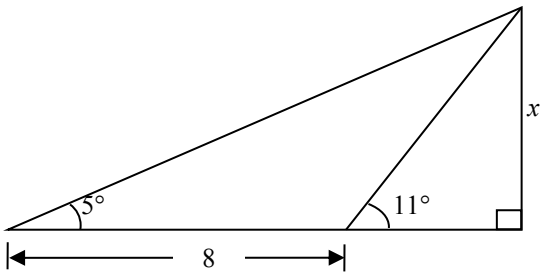
18. Find the length x.



19. Find the length x.

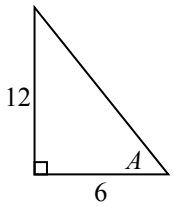


20. In traveling across flat land, you notice a mountain directly in front of you. Its angle of elevation (to the peak) is 5°. After you drive 8 miles closer to the mountain, the angle of elevation is 11°. Approximate the height of the mountain.



21. A plane is observed approaching your home and you assume that its speed is 570 miles per hour. The angle of elevation is 12° at one time and 63° one minute later. Approximate the altitude of the plane.

Answer to Odd Questions:

1. 

hypotenuse^2 = $12^2 + 6^2 = 160$ → hypotenuse = $\sqrt{180} = 6\sqrt{5}$

Therefore, $\sin(A) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{12}{6\sqrt{5}} = \frac{2}{\sqrt{5}}$

$\cos(A) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{6}{6\sqrt{5}} = \frac{1}{\sqrt{5}}$

$\tan(A) = \frac{\sin(A)}{\cos(A)} = \frac{\frac{2}{\sqrt{5}}}{\frac{1}{\sqrt{5}}} = 2$ or $\tan(A) = \frac{\text{opposite}}{\text{adjacent}} = \frac{12}{6} = 2$

$\sec(A) = \frac{1}{\cos(A)} = \frac{1}{\frac{1}{\sqrt{5}}} = \sqrt{5}$

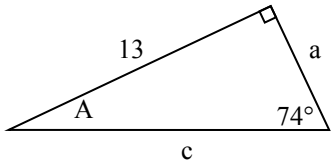
$\csc(A) = \frac{1}{\sin(A)} = \frac{1}{\frac{2}{\sqrt{5}}} = \frac{\sqrt{5}}{2}$

and $\cot(A) = \frac{1}{\tan(A)} = \frac{1}{2}$

3. $\cos A = \frac{24}{25}$ $\csc A = \frac{25}{7}$

$\sin A = \frac{7}{25}$ $\sec A = \frac{25}{24}$

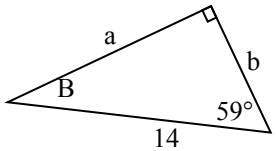
$\tan A = \frac{7}{24}$ $\cot A = \frac{24}{7}$

5. 

$\sin(74°) = \frac{13}{c}$ → $c = \frac{13}{\sin(74°)} \approx 13.524$

$\tan(74°) = \frac{13}{a}$ → $a = \frac{13}{\tan(74°)} \approx 3.728$

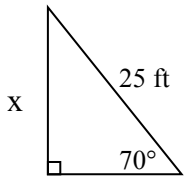
$A = 90° - 74° = 16°$

7. 

$B = 90° - 59° = 31°$

$\sin(B) = \sin(31°) = \frac{b}{14}$ → $b = 14 \sin(31°) \approx 7.211$

$\cos(B) = \cos(31°) = \frac{a}{14}$ → $a = 14 \cos(31°) \approx 12.000$



9. Let x (feet) be the height that the ladder reaches up.

Since $\sin(70°) = \frac{x}{25}$

So the ladder reaches up to x = 25 sin (70°) ≈ 23.4923 ft of the building.

11. Let y (miles) be the height of the building. Since $\tan(5°) = \frac{y}{2}$, the height of the building is y = 2tan (5°) mi ≈ 923.9 ft.

13. Let z_1 (feet) and z_2 (feet) be the heights of the upper and lower parts of the radio tower. We have

$\tan(40°) = \frac{z_1}{600} \Rightarrow z_1 = 600 \tan(40°)$ ft

$\tan(19°) = \frac{z_2}{600} \Rightarrow z_2 = 600 \tan(19°)$ ft

So the height of the tower is $z_1 + z_2 = 600 \tan(40°) + 600 \tan(19°) \approx 710.0563$ ft.

15. The height of the antenna = 500 tan (43°) - 500 tan (40°) ≈ 46.7077 ft.

17. $x = \frac{74\cos(37^\circ)}{\tan(33^\circ)} + 74\sin(37^\circ) \approx 135.5388$

19. $x = 157\cos(17^\circ) - \frac{157\sin(17^\circ)}{\tan(61^\circ)} \approx 124.7$

21. We have $\tan(12°) = \frac{a}{x+\frac{570}{60}} \Rightarrow a = \left(x + \frac{570}{60}\right) tan12$

$\tan(63°) = \frac{a}{x} \Rightarrow a = xtan63$

Therefore $a = \left(x + \frac{570}{60}\right) tan12 = xtan63$

$\Rightarrow x = \frac{\frac{570}{60} tan12}{tan63 - tan12}$

$\Rightarrow a = xtan63 = \frac{\frac{570}{60} tan12}{tan63 - tan12} * tan63 \approx 2.265$