

## HW 4.1.2: Simplifying and Verifying Trigonometric Expressions

Simplify each of the following to an expression involving a single trig function with no fractions.

1.)  $\csc(t) \tan(t)$

2.)  $\cos(t) \csc(t)$

3.)  $\frac{\sec(t)}{\csc(t)}$

4.)  $\frac{\sec(\theta)}{\tan(\theta)}$

5.)  $\frac{\cot(t)}{\csc(t)}$

6.)  $\frac{1 - \sin^2(t)}{\sin^2(t)}$

7.)  $\frac{\sin^2(t) + \cos^2(t)}{\cos^2(t)}$

8.)  $\frac{\sec(t) - \cos(t)}{\sin(t)}$

9.)  $\sin x (\tan x + \cot x)$

10.)  $(1 + \sin \alpha)(1 - \sin \alpha)$

Verify the identity.

$$11.) \frac{\sin^2(\theta)}{1 + \cos(\theta)} = 1 - \cos(\theta)$$

$$12.) \frac{1 + \cot(\alpha)}{\csc(\alpha)} = \sin(\alpha) + \cos(\alpha)$$

$$13.) \frac{\cos^2(\theta)}{1 + \sin(\theta)} = 1 - \sin(\theta)$$

$$14.) \frac{1 + \tan^2(b)}{\tan^2(b)} = \csc^2(b)$$

$$15.) \cos(x)(\sec(x) + \csc(x)) = 1 + \cot(x)$$

$$16.) \frac{(1 + \cos(A))(1 - \cos(A))}{\sin(A)} = \sin(A)$$

$$17.) \frac{\sin^4(\gamma) - \cos^4(\gamma)}{\sin(\gamma) - \cos(\gamma)} = \sin(\gamma) + \cos(\gamma)$$

$$18.) \frac{1 - \sin(t)}{\cos^2(t)} + \frac{1}{1 - \sin(t)} = 2 \sec^2(t)$$

$$19.) (\cos x - \sin x)^2 + (\cos x + \sin x)^2 = 2$$

$$20.) \frac{\sec^2 x - \tan^2 x + \tan x}{\sec x} = \sin x + \cos x$$

Selected Answers:

1.)  $\sec(t)$

3.)  $\tan(t)$

5.)  $\cos(t)$

7.)  $\sec^2(t)$

9.)  $\sec(x)$

$$11.) \frac{\sin^2(\theta)}{1 + \cos(\theta)} = \frac{1 - \cos^2(\theta)}{1 + \cos(\theta)} = \frac{(1 + \cos(\theta))(1 - \cos(\theta))}{1 + \cos(\theta)} = 1 - \cos(\theta)$$

$$15.) \cos(x)(\sec(x) + \csc(x)) = \cos(x) \left( \frac{1}{\cos(x)} + \frac{1}{\sin(x)} \right) = 1 + \frac{\cos(x)}{\sin(x)} = 1 + \cot(x)$$

$$17.) \frac{\sin^4(\gamma) - \cos^4(\gamma)}{\sin(\gamma) - \cos(\gamma)} = \frac{(\sin(\gamma) - \cos(\gamma))(\sin(\gamma) + \cos(\gamma))(\sin^2(\gamma) + \cos^2(\gamma))}{\sin(\gamma) - \cos(\gamma)} = \sin(\gamma) + \cos(\gamma)$$

$$\begin{aligned} & (\cos x - \sin x)^2 + (\cos x + \sin x)^2 \\ 19.) & = (\cos^2 x - 2\cos x \sin x + \sin^2 x) + (\cos^2 x + 2\cos x \sin x + \sin^2 x) \\ & = 2(\cos^2 x + \sin^2 x) = 2 \end{aligned}$$