

## HW 4.2.1.c: Solving Trigonometric Equations (using algebra)

Find all solutions on the interval  $0 \leq \theta < 2\pi$ .

1.  $2 \sin(\theta) = -1$       2.  $2 \sin(\theta) = \sqrt{3}$       3.  $2 \cos(\theta) = 1$       4.  $2 \cos(\theta) = -\sqrt{2}$

5.  $2 \sin\left(\frac{\pi}{4}x\right) = 1$       6.  $2 \sin\left(\frac{\pi}{3}x\right) = \sqrt{2}$       7.  $2 \cos(2t) = -\sqrt{3}$       8.  $2 \cos(3t) = -1$

9.  $\sin^2 x = \frac{1}{4}$       10.  $\cos^2 \theta = \frac{1}{2}$       11.  $\sec^2 x = 4$       12.  $3 \csc^2 t = 4$

13.  $\csc(2x) - 2 = 0$

14.  $\sec(2\theta) = 2$

15.  $2 \sin(x) \cos(x) = \cos(x)$

16.  $\sqrt{2} \sin(t) = 2 \cos(t) \sin(t)$

17.  $\sec(x) \sin(x) - 2 \sin(x) = 0$

18.  $\tan(x) \sin(x) - \sin(x) = 0$



19.  $\tan^3(x) = 3 \tan(x)$

20.  $\cos^3(t) = \cos(t)$

21.  $\tan^3(x) = \tan(x)$

22.  $\tan^5(x) - 9 \tan(x) = 0$

23.  $2 \sin^2 w + 3 \sin w + 1 = 0$

24.  $2 \cos^2 x + 3 \cos(x) + 1 = 0$

25.  $2 \cos^2 t + \cos t = 1$

26.  $2 \sin^2(\theta) + 3 \cos(\theta) - 3 = 0$

27.  $2 \cos^2(\theta) = 2 + \sin(\theta)$

28.  $4 \sin(x) \cos(x) + 2 \sin(x) - 2 \cos(x) - 1 = 0$

29.  $2 \sin(x) \cos(x) - \sin(x) + 2 \cos(x) - 1 = 0$

Selected Answers: (Teachers feel free to add more solutions)

1. On the interval  $0 \leq \theta < 2\pi$ ,  $\theta = \frac{7\pi}{6}$  or  $\theta = \frac{11\pi}{6}$ . Without this interval,  $\theta = \frac{7\pi}{6} \pm 2n\pi$  or  $\theta = \frac{11\pi}{6} \pm 2n\pi$

3. On the interval  $0 \leq \theta < 2\pi$ ,  $\theta = \frac{\pi}{3}$  or  $\theta = \frac{5\pi}{3}$ . Without this interval,  $\theta = \frac{\pi}{3} \pm 2n\pi$  or  $\theta = \frac{5\pi}{3} \pm 2n\pi$

5. On the interval  $0 \leq \theta < 2\pi$ ,  $x = \frac{2}{3}$  or  $\frac{10}{3}$ . Without this interval,  $\theta = \frac{2}{3} \pm 8n$  or  $\theta = \frac{10}{3} \pm 8n$

7. On the interval  $0 \leq \theta < 2\pi$ ,  $t = \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}$ .

9. On the interval  $0 \leq \theta < 2\pi$ , this occurs at  $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

11. On the interval  $0 \leq \theta < 2\pi$ , this occurs at  $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

13.  $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

15. On the interval  $0 \leq \theta < 2\pi$ ,  $\cos x = 0$  at  $x = \frac{\pi}{2}$  and  $x = \frac{3\pi}{2}$ , and  $\sin x = \frac{1}{2}$  at  $x = \frac{\pi}{6}$  and  $x = \frac{5\pi}{6}$ .

$$\sin x = 0 \quad \sec x = 2$$

17.  $x = 0, \pi$      $\cos x = \frac{1}{2}$   
 $x = \frac{\pi}{3}, \frac{5\pi}{3}$

19. On the interval  $0 \leq \theta < 2\pi$ ,  $\tan x = 0$  at  $x = 0$  and  $x = \pi$ . Similarly,  $\tan x = \sqrt{3}$  at  $x = \frac{\pi}{3}$  and  $x = \frac{4\pi}{3}$ , and  $\tan x = -\sqrt{3}$  at  $x = \frac{2\pi}{3}$  and  $x = \frac{5\pi}{3}$ .

21. On the interval  $0 \leq \theta < 2\pi$ ,  $\tan x = 0$  at  $x = 0$  and  $x = \pi$ . Similarly,  $\tan x = 1$  at  $x = \frac{\pi}{4}$  and  $x = \frac{5\pi}{4}$ . Finally,  $\tan x = -1$  for  $x = \frac{3\pi}{4}$  and  $x = \frac{7\pi}{4}$ .

23. On the interval  $0 \leq \theta < 2\pi$ ,  $\sin w = -\frac{1}{2}$  at  $w = \frac{7\pi}{6}$  or  $w = \frac{11\pi}{6}$ . Similarly,  $\sin w = -1$  at  $w = \frac{3\pi}{2}$



25. On the interval  $0 \leq \theta < 2\pi$ ,  $\cos t = \frac{1}{2}$  at  $t = \frac{\pi}{3}$  or  $t = \frac{5\pi}{3}$ . Similarly,  $\cos t = -1$  at  $t = \pi$ .

$$\sin \theta = 0 \quad 2 \sin \theta + 1 = 0$$

$$27. \theta = 0, \pi \quad \sin \theta = -\frac{1}{2}$$

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$

29. Either  $2\cos(x) - 1 = 0$  or  $\sin(x) + 1 = 0$ . This leads to  $x = \frac{\pi}{3}$ ,  $x = \frac{5\pi}{3}$  (for  $\cos x = \frac{1}{2}$ ) and  $x = \frac{3\pi}{2}$ , (for  $\sin x = -1$ ).