# HW 4.3.6: Inverse Trigonometry

Sketch a graph of each function and identify the domain and range.

1. $f(x) = \sin x$

2. $f(x) = \sin^{-1} x$

3. $f(x) = \cos x$

4. $f(x) = \cos^{-1} x$

5. $f(x) = \tan x$

6. $f(x) = \tan^{-1} x$

Evaluate. Use the domain and range given in the table 4.3.6-1 in Exploration 4.3.6.

7. $\sin^{-1}\left(\frac{1}{2}\right)$

8. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

9. $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

10. $\sin^{-1}(-1)$

11. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

12. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

13. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

14. $\cos^{-1}\left(-\frac{1}{2}\right)$

15. $\tan^{-1}(-1)$

16. $\tan^{-1}(-\sqrt{3})$

17. $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

18. $\tan^{-1}(1)$

Evaluate the following expressions.

19. $\sin^{-1}\left(\cos\left(\frac{\pi}{2}\right)\right)$

20. $\cos^{-1}\left(\sin\left(\frac{\pi}{3}\right)\right)$

21. $\sin^{-1}\left(\cos\left(\frac{2\pi}{3}\right)\right)$

22. $\cos^{-1}\left(\sin\left(\frac{\pi}{4}\right)\right)$

23. $\cos\left(\sin^{-1}\left(\frac{3}{11}\right)\right)$

24. $\sin\left(\cos^{-1}\left(\frac{2}{7}\right)\right)$

25. $\cos\left(\tan^{-1}(3)\right)$

26. $\tan\left(\sin^{-1}\left(\frac{2}{5}\right)\right)$

Find a simplified expression for each of the following.

27. $\cos\left(\sin^{-1}(x)\right)$, for $-1 \le x \le 1$

28. $\sin\left(\cos^{-1}(x)\right)$, for $-1 \le x \le 1$

29. $\sin\left(\cos^{-1}\left(\frac{x}{4}\right)\right)$, for $-4 \le x \le 4$

30. $\tan\left(\cos^{-1}\left(\frac{x}{6}\right)\right)$, for $-6 \le x \le 6$

31. $\sin\left(\tan^{-1}\left(5x\right)\right)$

32. $\cos\left(\tan^{-1}\left(2x\right)\right)$

33. $\tan\left(\sin^{-1}\left(x+1\right)\right)$, for $x \ne 0, x \ne 2$

34. $\cos\left(\sin^{-1}\left(\frac{3x}{4}\right)\right)$, for $x \le \frac{4}{3}$

<u>Selected Answers</u>:

7. $\frac{\pi}{6}$.

9. $\frac{\pi}{4}$.

11. $\frac{\pi}{6}$.

13. $\frac{5\pi}{6}$.

15. $-\frac{\pi}{4}$.

17. $\frac{\pi}{6}$.

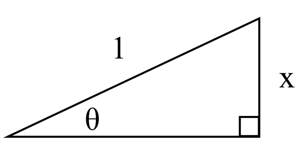
19. 0

21. $-\frac{\pi}{6}$.

23. $\frac{4\sqrt{7}}{11}$

25. $\frac{1}{\sqrt{10}}$.

27. Let θ = $\sin^{-1}(x)$, then sin (θ) = $\frac{x}{1} = \frac{\text{opposite}}{\text{hypotenuse}}$




Using the Pythagorean Theorem, we can find the opposite of the triangle:

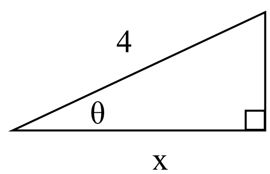
$$x^2 + \text{adjacent}^2 = 1^2 = 1$$

$$\text{adjacent}^2 = 1 - x^2$$

adjacent = $\sqrt{1 - x^2}$

Therefore, $\cos(\sin^{-1}(x)) = \cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\sqrt{1 - x^2}}{1} = \sqrt{1 - x^2}$.

29. Let $\theta = \cos^{-1}\left(\frac{x}{4}\right)$, then $\cos(\theta) = \frac{x}{4} = \frac{\text{adjacent}}{\text{hypotenuse}}$




Using the Pythagorean Theorem, we can find the opposite of the triangle:

$x^2 + \text{opposite}^2 = 4^2 = 16$

$\text{opposite}^2 = 16 - x^2$

opposite = $\sqrt{16 - x^2}$

Therefore, $\sin\left(\cos^{-1}\left(\frac{x}{4}\right)\right) = \sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\sqrt{16 - x^2}}{4}$.

31. $\sin(\tan^{-1}(5x)) = \sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{5x}{\sqrt{25x^2 + 1}}$.

33. $\tan(\sin^{-1}(x + 1)) = \tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} = \frac{x+1}{\sqrt{-x^2-2x}}$.