The University of Texas at Austin
OnRamps

HW 4.5.2.a: Sum and Difference Formulas

Use the sum and difference formulas to find an exact value for each of the following.

1. $\sin(105°)$ 2. $\sin(135°)$ 3. $\cos(195°)$ 4. $\cos(315°)$

5. $\sin\left(\frac{7\pi}{12}\right)$ 6. $\sin\left(\frac{\pi}{12}\right)$ 7. $\cos\left(\frac{5\pi}{12}\right)$ 8. $\cos\left(\frac{11\pi}{12}\right)$

Rewrite in terms of $\sin(x)$ and $\cos(x)$.

9. $\cos\left(x+\frac{11\pi}{6}\right)$ 10. $\cos\left(x-\frac{\pi}{4}\right)$ 11. $\sin\left(x-\frac{7\pi}{6}\right)$ 12. $\sin\left(x+\frac{4\pi}{3}\right)$

Solve each equation for all solutions.

13. $\sin(3x)\cos(6x)-\cos(3x)\sin(6x)=\frac{\sqrt{2}}{2}$ 14. $\sin(11x)\cos(6x)-\cos(11x)\sin(6x)=\frac{\sqrt{3}}{2}$

15. $\cos(2x)\cos(x)+\sin(2x)\sin(x)=1$

16. $\cos(5x)\cos(3x)-\sin(5x)\sin(3x)=\frac{\sqrt{3}}{2}$

17. How could you evaluate $\tan\left(\frac{13\pi}{12}\right)$ if you did not know the sum and difference formula for tangent?

18. Prove the sine and cosine cofunction identities using the sum and difference formulas.

Using your answers from 1-8 and the even/odd identities, find the following:

19. $\sin(-105°)$

20. $\sin(-135°)$

21. $\cos(-195°)$

22. $\cos(-315°)$

23. $\sin\left(-\frac{7\pi}{12}\right)$

24. $\sin\left(-\frac{\pi}{12}\right)$

25. $\cos\left(-\frac{5\pi}{12}\right)$

26. $\cos\left(-\frac{11\pi}{12}\right)$

Selected Answers:

1. $\sin(105°) = \sin(60°+45°) = \sin(60°)\cos(45°)+\cos(60°)\sin(45°) = \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}+\frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$

3. $\cos(195°) = \cos(150° + 45°) = \cos(150°)\cos(45°)\text{-}\sin(150°)\sin(45°) = \frac{-(\sqrt{2}+\sqrt{6})}{4}$

5. $\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{\pi}{4}+\frac{\pi}{3}\right) = \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) + \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{2}+\sqrt{6}}{4}$

7. $\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{6}+\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{6}-\sqrt{2}}{4}$

9. $\cos\left(x+\frac{11\pi}{6}\right) = \cos(x)\cos\left(\frac{11\pi}{6}\right) - \sin(x)\sin\left(\frac{11\pi}{6}\right) = \frac{\sqrt{3}}{2}\cos(x) + \frac{1}{2}\sin(x)$

11. $\sin\left(x-\frac{7\pi}{6}\right) = \sin(x)\cos\left(\frac{7\pi}{6}\right) - \cos(x)\sin\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}\sin(x) + \frac{1}{2}\cos(x)$

13. $\sin(3x)\cos(6x) - \cos(3x)\sin(6x) = \frac{\sqrt{2}}{2}$

    $\sin(3x-6x) = \frac{\sqrt{2}}{2}$

    $\sin(-3x) = \frac{\sqrt{2}}{2}$

    $-\sin(3x) = \frac{\sqrt{2}}{2}$, $\sin(3x) = -\frac{\sqrt{2}}{2}$

    $3x = \frac{5\pi}{4} + 2\pi k$ or $3x = \frac{7\pi}{4} + 2\pi k$, where k is an integer

    $x = \frac{5\pi}{12} + \frac{2\pi}{3}k$ or $x = \frac{7\pi}{12} + \frac{2\pi}{3}k$, where k is an integer

15. $\cos(2x)\cos(x) + \sin(2x)\sin(x) = 1$

    $\cos(2x - x) = 1$

    $x = 0 + 2\pi k$, where k is an integer

19. $-\frac{(\sqrt{6}+\sqrt{2})}{4}$

21. $\frac{-(\sqrt{2}+\sqrt{6})}{4}$

23. $\frac{-(\sqrt{2}+\sqrt{6})}{4}$

25. $\frac{\sqrt{6}-\sqrt{2}}{4}$