HW 4.5.2.b: Double Angle Formulas

1. If $\sin(x)=\frac{1}{4}$ and x is in quadrant I, then find exact values for (without solving for x):

   a. $\sin(2x)$ b. $\cos(2x)$ c. $\tan(2x)$

2. If $\cos(x)=\frac{3}{5}$ and x is in quadrant I, then find exact values for (without solving for x):

   a. $\sin(2x)$ b. $\cos(2x)$ c. $\tan(2x)$

Evaluate each expression without a calculator.

3. $\cos^2(22.5°)-\sin^2(22.5°)$

4. $2\cos^2(67.5°)-1$

5. $2\sin(112.5^\circ)\cos(112.5^\circ)$

6. $\frac{2\tan(75^\circ)}{1-\tan^2(75^\circ)}$

Simplify each expression.

7. $\cos^2(9x)-\sin^2(9x)$

8. $\cos^2(6x)-\sin^2(6x)$

9. $4\sin(8x)\cos(8x)$

10. $6\sin(5x)\cos(5x)$

Solve for all solutions on the interval $[0,2\pi)$.

11. $6\sin(2t)+6\sin(t)=0$

12. $\sin(2t)+\sqrt{3}\cos(t)=0$

13. $\cos(2\theta)=6\cos^2(\theta)-4$

14. $2\cos^2 x-2\cos 2x=1$

15. $\sin(2t)=\cos(t)$

16. $\cos(2t)=\sin(t)$

17. $\cos(6x)-\cos(3x)=0$

18. $\sin(2x)\sin(x)=\cos(x)$

Prove the identity, Manipulate the left side to equal the right side.

19. $(\sin t-\cos t)^2=1-\sin(2t)$

20. $(\sin^2 x-1)^2=\cos(2x)+\sin^4 x$

21. $\cos^4 x-\sin^4 x=\cos(2x)$

22. $\dfrac{\sin(2\theta)}{1+\cos(2\theta)}=\tan(\theta)$

23. $cos^2\left(\theta-\frac{\pi}{2}\right)+cos^2(\theta)=1$

24. $sin\left(\theta-\frac{\pi}{2}\right)tan(\theta)=sin(\theta)$

Selected Answers:

1. a. $\sin(2x) = 2\sin x \cos x$

To find $\cos\ x$:

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \frac{1}{16} = \frac{15}{16}$

$\cos x = \pm\sqrt{\frac{15}{16}}$ Note that we need the positive root since we are told x is in quadrant 1.

$\cos x = \frac{\sqrt{15}}{4}$

So: $\sin(2x) = 2\left(\frac{1}{4}\right)\left(\frac{\sqrt{15}}{4}\right) = \frac{\sqrt{15}}{8}$

b. $\cos(2x) = 2\cos^2 x - 1$

$= (2\left(\frac{15}{16}\right)) - 1 = \frac{30-16}{16} = \frac{7}{8}$

c. $\tan(2x) = \frac{\sin(2x)}{\cos(2x)} = \frac{\sqrt{15}}{8}/\frac{7}{8} = \frac{\sqrt{15}}{7}$

3. $\cos^2 x - \sin^2 x = \cos(2x)$, so $\cos^2(22.5°) - \sin^2(22.5°) = \cos(45°) = \frac{\sqrt{2}}{2}$

5. $2\sin(x)\cos(x) = \sin(2x)$, so $2\sin(112.5°)\cos(112.5°) = \sin(225°) = -\frac{\sqrt{2}}{2}$

7. $\cos^2(9x) - \sin^2(9x) = \cos\big(2(9x)\big) = \cos(18x)$

9. $4\sin(8x)\cos(8x) = 2\,(2\sin(8x)\cos(8x)) = 2\sin(16x)$

11. $6\sin(2t) + 6\sin t = 6 \cdot 2\sin t\cos t + 6\sin t = 6\sin t\,(2\cos t + 1)$, so we can solve

$6\sin t = 0$ or $\cos t = -1/2$

$t = 0, \pi$ or $t = \frac{2\pi}{3}, \frac{4\pi}{3}$

13. $\cos(2\theta) = 6\cos^2\theta - 4$, $2\cos^2\theta - 1 = 6\cos^2\theta - 4$

$3 = 4\cos^2\theta$, $\pm\sqrt{3}/2 = \cos\theta$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

15. $\sin(2t) = \cos t$

$2 \sin t \cos t = \cos t$

$2 \sin t \cos t - \cos t = 0$

$\cos t\,(2 \sin t - 1) = 0$

$\cos t = 0$ or $2 \sin t - 1 = 0$

If $\cos t = 0$, then $t = \frac{\pi}{2}$ or $\frac{3\pi}{2}$. If $2 \sin t - 1 = 0$, then $\sin t = \frac{1}{2}$, so $t = \frac{\pi}{6}$ or $\frac{5\pi}{6}$. So $t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}$ or $\frac{5\pi}{6}$.

17. $\cos(6x) - \cos(3x) = 0$

$2\cos^2(3x) - 1 - \cos(3x) = 0$

$2\cos^2(3x) - \cos(3x) - 1 = 0$

$(2\cos(3x) + 1)(\cos(3x)) - 1 = 0$

$\cos(3x) = -\frac{1}{2}$ or 1

Since we need solutions for x in the interval $[0, 2\pi)$, we will look for all solutions for $3x$ in the interval $[0, 6\pi)$. If $\cos(3x) = -\frac{1}{2}$, then there are two possible sets of solutions. First, $3x = \frac{2\pi}{3} + 2\pi k$ where $k = 0, 1,$ or 2, so $x = \frac{2\pi}{9} + \frac{2\pi k}{3}$ where $k = 0, 1,$ or 2. Second, $3x = \frac{4\pi}{3} + 2\pi k$ where $k = 0, 1,$ or 2, so $x = \frac{4\pi}{9} + \frac{2\pi k}{3}$ where $k = 0, 1,$ or 2 If $\cos(3x) = 1$, then $3x = 2\pi k$ where $k = 0, 1,$ or 2, so $x = \frac{2\pi k}{3}$ where $k = 0, 1,$ or 2.

23. $cos^2\left(\theta - \frac{\pi}{2}\right) + cos^2(\theta) = 1$, substitute the cofunction identity $sin(\theta) = cos\left(\theta - \frac{\pi}{2}\right)$

$sin^2(\theta) + cos^2(\theta) = 1$

$1 = 1$