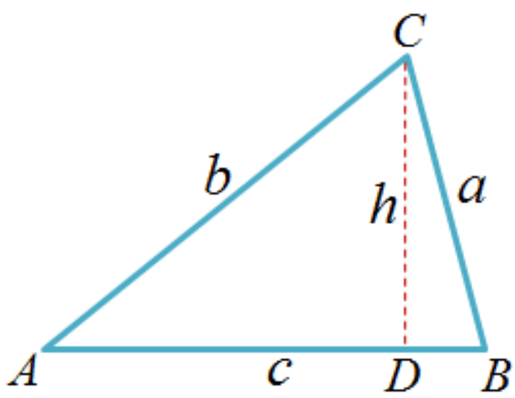
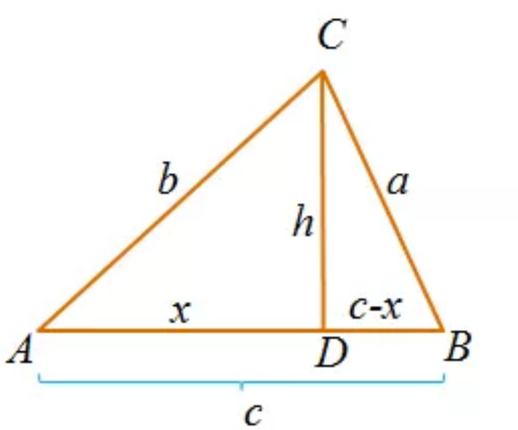
HW 4.5.3: Law of Sines and Cosines

Derive the Law of Sines using the diagram below.

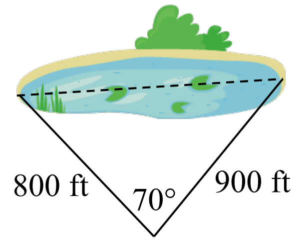


Derive the Law of Cosines using the diagram below.

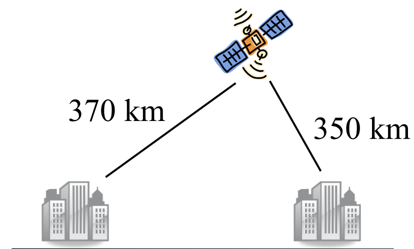


Homework Problems

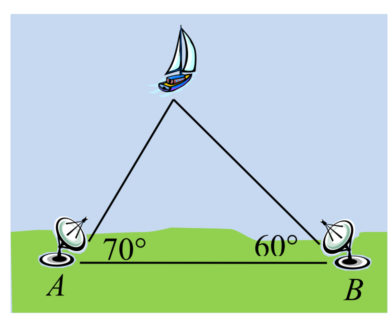
1. To find the distance across a small lake, a surveyor has taken the measurements shown. Find the distance across the lake.



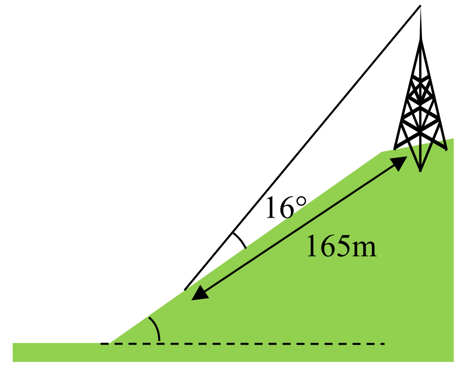
2. The distance between two cities is 150 kilometers. Each city communicates with the satellite as shown in the picture. What is the angle of elevation the signal makes with the ground for each city?



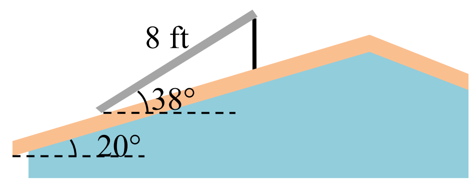
3. To determine how far a boat is from shore, two radar stations 500 feet apart determine the angles out to the boat, as shown. Find the distance of the boat from the station *A*.



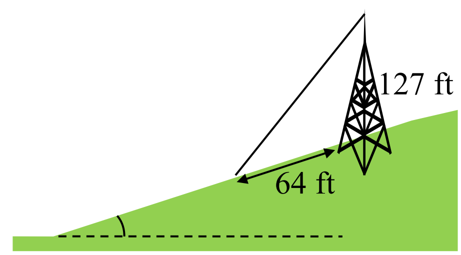
4. A communications tower is located at the top of a steep hill, as shown. A guy-wire is to be attached to the top of the tower and to the ground, 165 m downhill from the base of the tower. The height of the tower is 80 m, how long is the guy-wire?



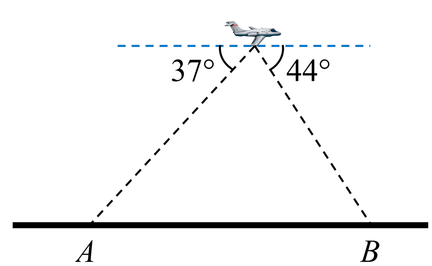
5. The roof of a house is at a 20° angle. An 8 foot solar panel is to be mounted on the roof, and should be angled 38° relative to the horizontal for optimal results. How long does the vertical support holding up the back of the panel need to be?



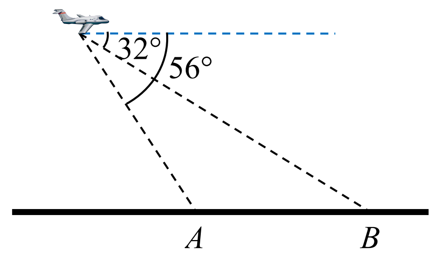
6. A 127 foot tower is located on a hill. A guy-wire is to be attached to the top of the tower and anchored at a point 64 feet downhill from the base of the tower. The guy-wire is 175 feet long. Find the angle the tower makes with the hill.



7. A pilot is flying over a straight highway. He determines the angles of depression to two mileposts, 6.6 km apart, to be 37° and 44°, as shown in the figure. Find the distance of the plane from point *A*, and the elevation of the plane.



8. A pilot is flying over a straight highway. He determines the angles of depression to two mileposts to be 32° and 56°, as shown in the figure. The pilot is flying at an altitude of 25,000 ft. Find the distance of the plane from point *A* to *B*.

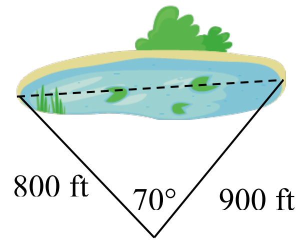


9. To estimate the height of a building, two students find the angle of elevation from a point (at ground level) down the street from the building to the top of the building is 39°. From a point that is 300 feet closer to the building, the angle of elevation (at ground level) to the top of the building is 50°. If we assume that the street is level, use this information to estimate the height of the building.

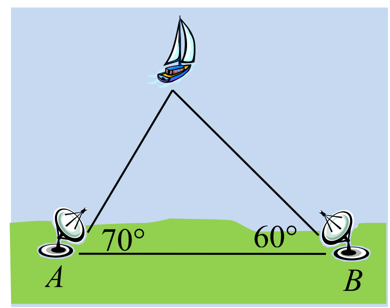
10. A pilot flies in a straight path for 1 hour 30 min. She then makes a course correction, heading 10 degrees to the right of her original course, and flies 2 hours in the new direction. If she maintains a constant speed of 680 miles per hour, how far is she from her starting position?

Selected Answers:

1. Because the angle corresponds to neither of the given sides it is easiest to first use Law of Cosines, to find the third side. According to Law of Cosines $d^2 = 800^2 + 900^2 - 2(800)(900)\cos(70)$ where d is the distance across the lake. $d = 978.51$ft



3. The sum of all angles in a triangle must be 180°, so $180° = 70° + 60° + \theta, and\ \theta = 50°$.



Knowing this angle allows us to use Law of Sines to find d_A.

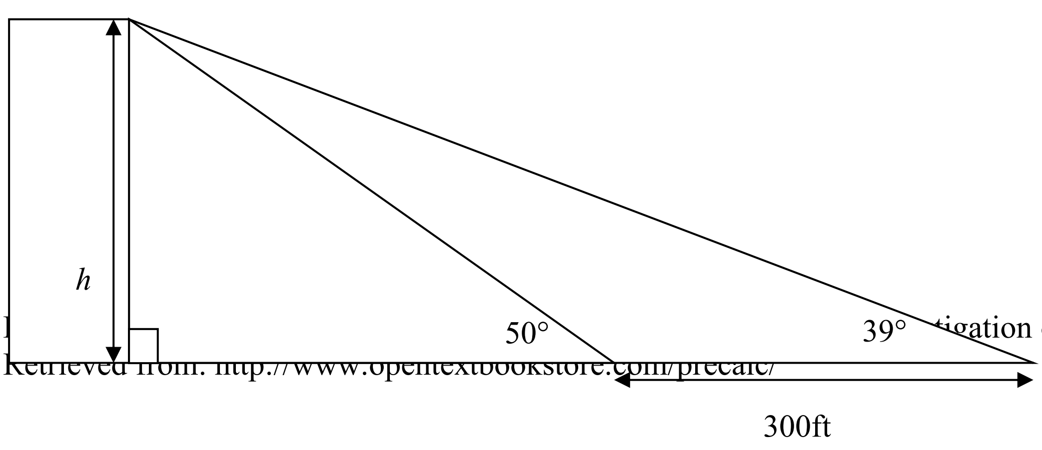
According to Law of Sines $\frac{\sin(50)}{500} = \frac{\sin(60)}{d_A} \Rightarrow d_A = \frac{500\sin(60)}{\sin(50)} \approx 565.26ft$.

7. Using the relationship between alternate interior angles, the angle at A inside the triangle is 37°, and the angle at B inside the triangle is 44°.

Because there is only one side given, it is best to use Law of Sines to solve for d. According to Law of Sines, $\frac{\sin(44)}{d} = \frac{\sin(99)}{6.6}$, so $d = \frac{6.6\sin(44)}{\sin(99)} \approx 4.64$ km.

9. Assuming the building is perpendicular with the ground, this situation can be drawn as two triangles.

Let h = the height of the building. Let x = the distance from the first measurement to the top of the building.



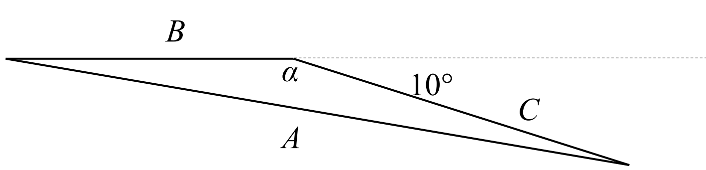
In order to find h, we need to first know the length of one of the other sides of the triangle. x can be found using Law of Sines and the triangle on the right.

The angle that is adjacent to the angle measuring 50° has a measure of 130°, because it is supplementary to the 50° angle. The angle of the top of the right hand triangle measures 11° since all the angles in the triangle have a sum of 180°.

According to Law of Sines, $\frac{\sin(39)}{x} = \frac{\sin(11)}{300ft}$, so $x = 989.45$ft.

Finding the value of h only requires trigonometry. $h = (989.45 \text{ ft})\sin(50) \approx 757.96$ ft.

10. Because the given information tells us two sides and information relating to the angle opposite the side we need to find, Law of Cosines must be used.



The angle α is supplementary with the 10° angle, so $\alpha = 180° - 10° = 170°$.

From the given information, the side lengths can be found:

$B = 1.5 \text{ hours} \cdot \frac{680 \text{ miles}}{1 \text{ hour}} = 1020 \text{ miles}.$

$C = 2 \text{ hours} \cdot \frac{680 \text{ miles}}{1 \text{ hour}} = 1360 \text{ miles}.$

According to Law of Cosines: so $A^2 = (1020)^2 + (1360\,)^2 - 2(1020)(1360\,)\cos(170)$. Solving for A gives $A \approx 2{,}371.13$ miles.