The University of Texas at Austin
OnRamps

HW 5.1.2: More Rational Functions

Part A: Vertical Asymptotes

Using technology (Desmos suggested) to explore these rational function characteristics.

1. Type this equation in Desmos: $f(x)=\dfrac{(x+1)^a}{(x-1)(x+2)}$

In Desmos: Click the "add slider: a," which will add a second line with "a=1." Click beside the 1 and an interval and a step will show up. Input _1_≤ a≤ _3_ Step: _1_. Hit "enter" or select the play triangle, and then you can click on the bar to stop the animation to view the "a" of choice.

Describe what happens to the graph when:

a = 1:

a = 2:

a = 3:

2. Type this equation in Desmos: $g(x)=\dfrac{(x+1)}{(x-1)^b(x+2)}$

Follow the same directions as above to use the slider.

Describe what happens to the graph when:

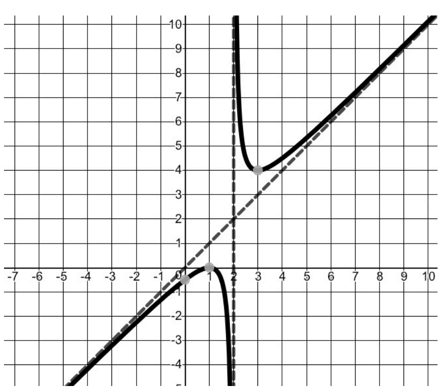
a = 1:

a = 2:

a = 3:

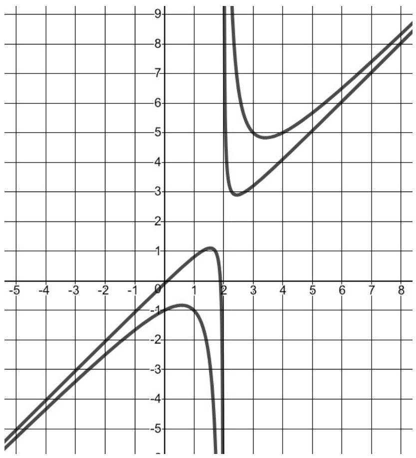
Part B: Slant Asymptotes

3. After performing division, $f\left(x\right)=\dfrac{ax^2+bx+c}{dx+e}$ can be written in the form $y=ax+b+\dfrac{c}{x-d}$. Identify which asymptotes $x-d$ and $ax+b$ will create.



4. Using technology (Desmos suggested) create a rational function in the form $y = ax + b + \frac{c}{x-d}$.

   Keeping c as a slider and constants for a, b, and d, determine the effect of a small c value (close to zero) compared to a large c value, will have on the graph.



5. What effect does a negative c have on the graph?

6. Which of the following functions will have a slant asymptote, why?

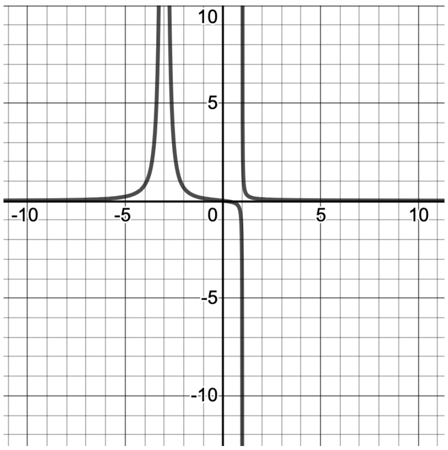
$$f(x) = \frac{x+2}{x^2 - x - 2} \quad \text{or} \quad f(x) = \frac{x^2 - x - 2}{x+2}$$

7. Sketch a possible graph for the function from 6 that creates the slant asymptote.

8. Summarize your findings in Part B of this assignment with respect to behavior around the slant asymptotes.

Create a possible function that could create the graph. Explain your thinking for each part.

9.



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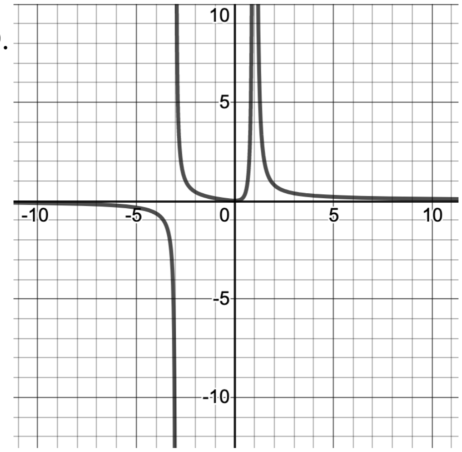

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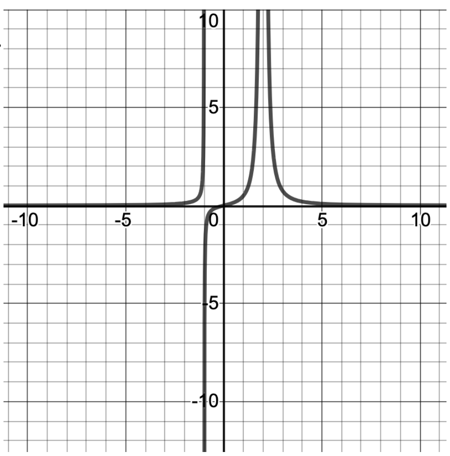

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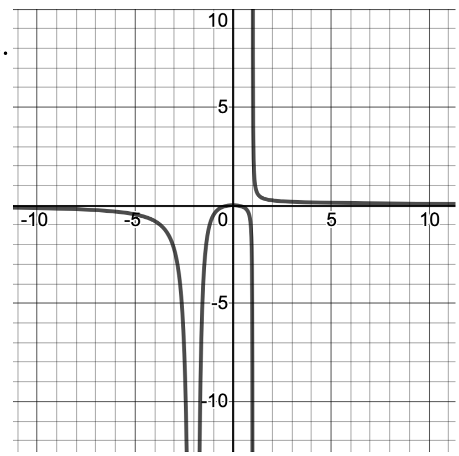

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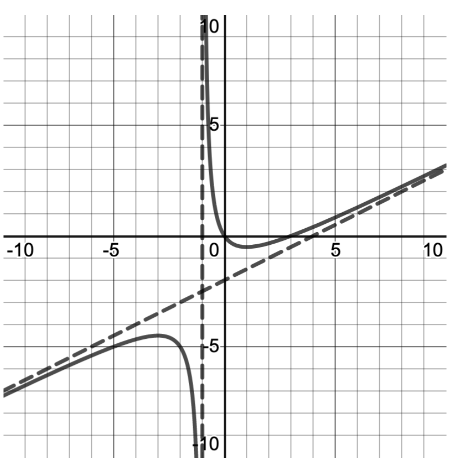

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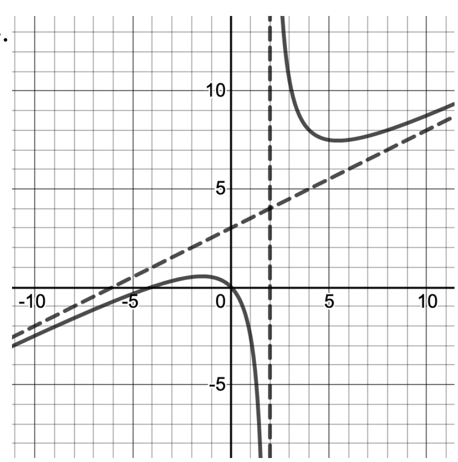

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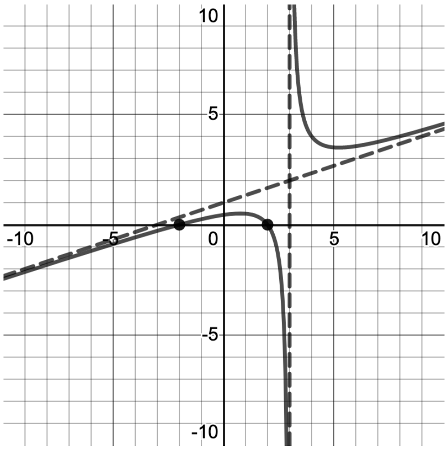

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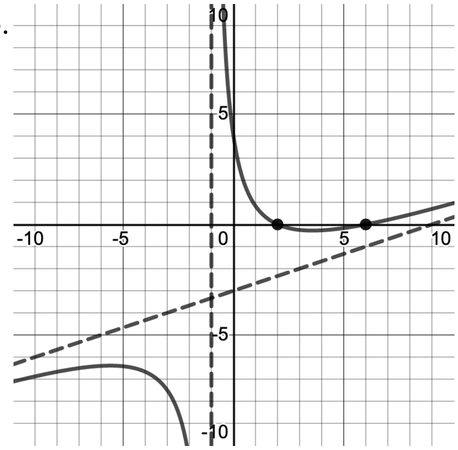
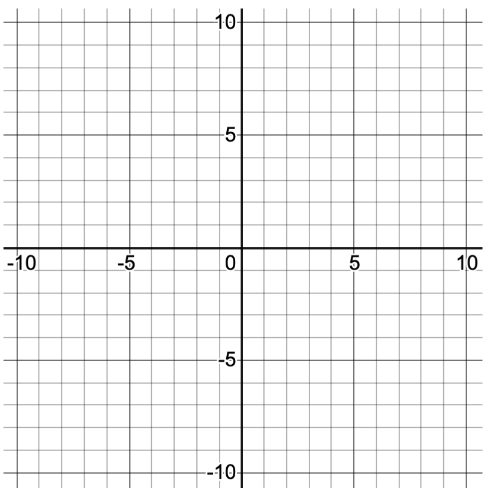
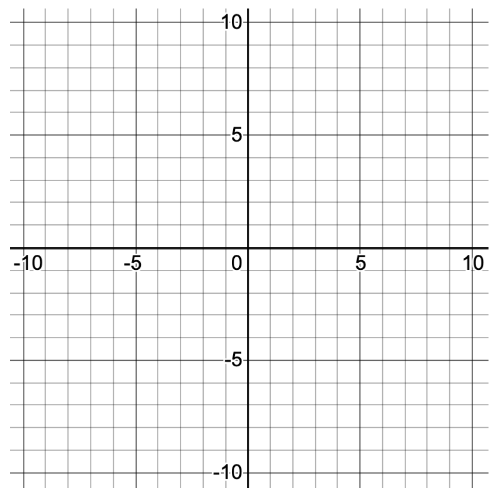


For each function, find the x-intercepts, y-intercept, vertical asymptotes, end behavior (horizontal or slant asymptotes), and holes if applicable. Use that information to sketch the graph.

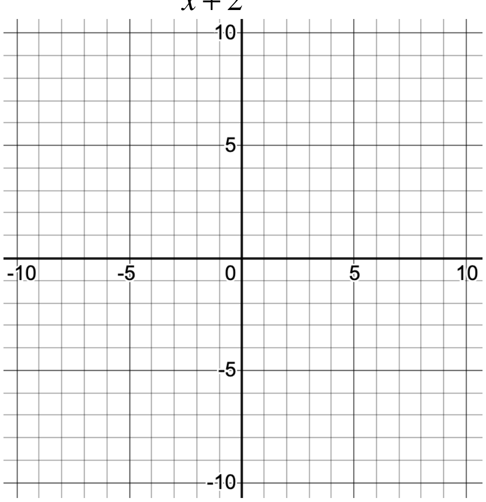
17. $k(x)=\dfrac{x^2-1}{x-2}$



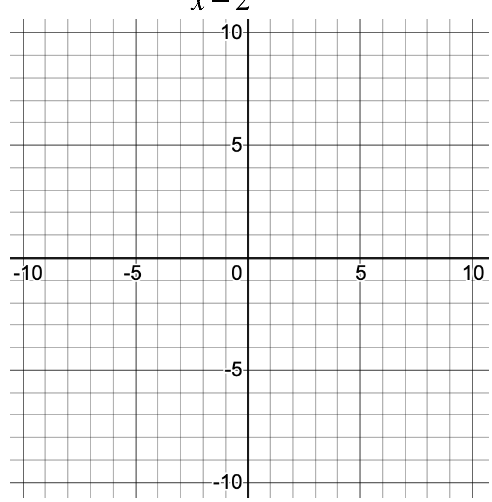
18. $m(x)=\dfrac{x^2+2x}{x-1}$



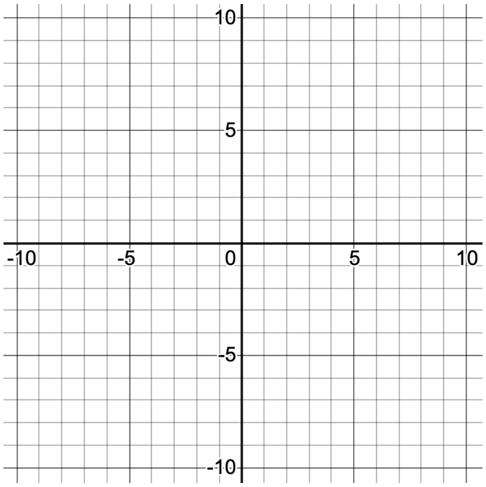
19. $p(x)=\dfrac{x^2+x-6}{x+2}$



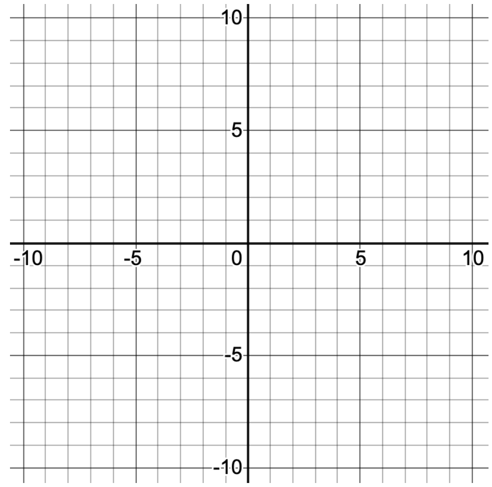
20. $q\left(x\right)=\dfrac{x^2-x-12}{x-2}$



21. $s(x)=\dfrac{(x+2)(x+3)(x-1)}{3(x+3)(x-2)}$



22. $r(x)=\dfrac{(x+3)(x+2)(x+1)}{2(x+2)(x+4)}$



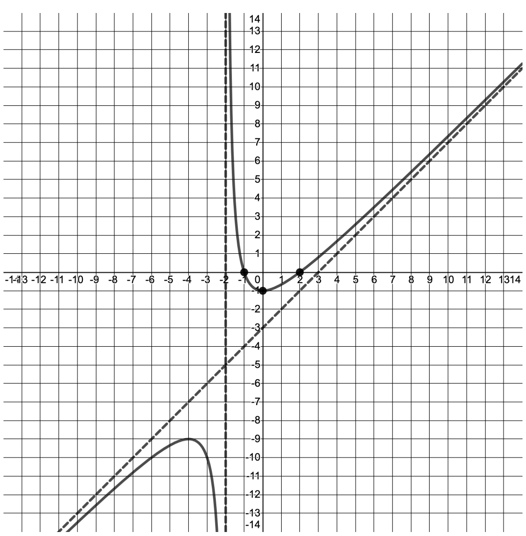
Selected Answers:

1. a=1: the graph crosses the x-intercept at x=-1
   a=2: the graph crosses the x-intercept and bounces off the x-axisYou do not need to re-write the problems.
   a=3: the graph crosses the x-intercept with a wave, and a slant asymptote occurs

3. x-d creates the vertical asymptote at x=d; ax+b creates the slant asymptote at y=ax+b

5. with a negative c, the graph's branches reflect over the slant asymptote

7.



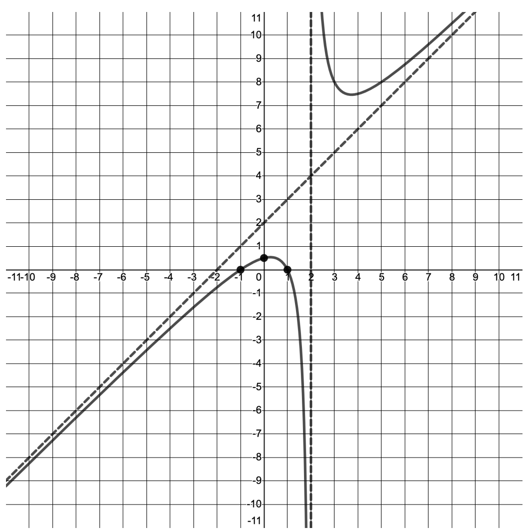
9. $f(x)=\dfrac{x}{(x+3)^2(x-1)}$; numerator is x because x-int is at (0, 0). Vertical asymptotes at x=3 and x=1, denominator has a squared on the (x-3) factor because the graph approaches infinity from the left and right of x=3

11. $f(x)=\dfrac{x}{(x+1)(x-2)^2}$; numerator is x because x-int is at (0, 0). Vertical asymptotes at x=-1 and x=2 create factors of (x+1) and (x-2) in the denominator. The (x-2) factor is squared because the graph approaches infinity from the left and right of x=2.

13. $f(x)=\dfrac{x(x-3)}{2(x+1)}$; numerator has factors x and (x-3) because those create the x-intercepts; denominator has factor of (x+1) because the graph has a vertical asymptote at x=-1; the slant asymptote exists because the numerator is one degree higher than the denominator. The slant asymptote has a slope of ½, so the 2 is needed in the denominator.

15. $f(x)=\dfrac{(x+2)(x-2)}{3(x-3)}$; numerator has factors (x+2) and (x-2) because those create the x-intercepts; donominator has factor of (x-3) because the graph has a vertical asymptote at x=3; the slant asymptote exists because the numerator is one degree higher than the denominator. The slant asymptote has a slope of 1/3, so the 3 is needed in the denominator.

17.



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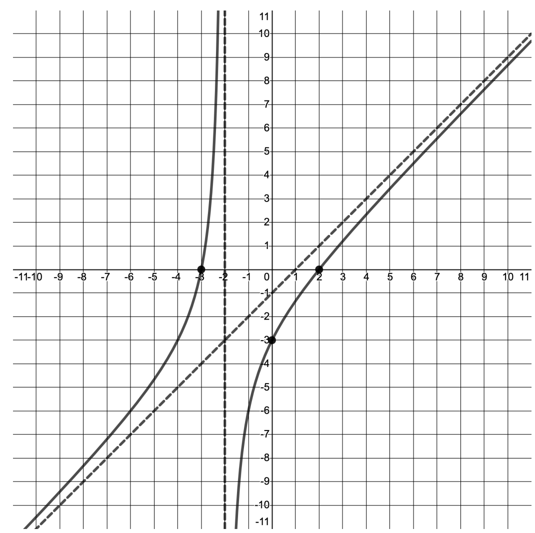

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