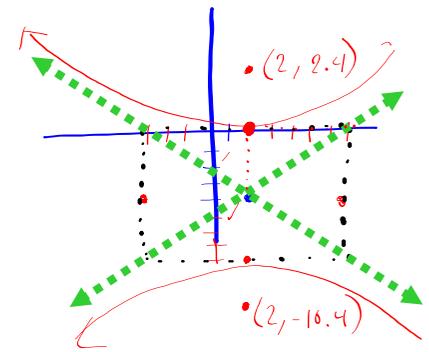7. $25y^2 - 16x^2 + 200y + 64x - 64 = 0$

$25y^2 + 200y - 16x^2 + 64x = 64$

$25(y^2 + 8y + \boxed{16}) - 16(x^2 - 4x + \boxed{4}) = 64 + \boxed{400} + \boxed{64}$

$$\frac{25(y+4)^2}{400} - \frac{16(x-2)^2}{400} = \frac{400}{400}$$

$$\frac{(y+4)^2}{16} - \frac{(x-2)^2}{25} = 1$$

CENTER: $(2, -4)$

VERTICES: $(2, 0)$, $(2, -8)$

FOCI: $(2, 2.4)$, $(2, -10.4)$

ASYMPTOTES: $y = \frac{4}{5}x - \frac{28}{5}$

$y = -\frac{4}{5}x - \frac{12}{5}$

$(2, 2.4)$

$(2, -10.4)$

$c^2 = a^2 + b^2$

$c^2 = 16 + 25$

$\sqrt{c^2} = \sqrt{41}$

$c \approx 6.4$

$y = \frac{4}{5}x + b$

$-4 = \frac{4}{5}(2) + b$

$-4 = \frac{8}{5} + b$

$-\frac{8}{5} \quad -\frac{8}{5}$

$y = -\frac{4}{5}x + b$

$-4 = -\frac{4}{5}(2) + b$

$-4 = -\frac{8}{5} + b$

$+\frac{8}{5} \quad +\frac{8}{5}$

$b = -\frac{28}{5} \qquad b \frac{(5)}{(5)} \frac{-4}{1} - \frac{8}{5}$

$b = -\frac{12}{5} \qquad b \frac{(5)}{(5)} \frac{-4}{1} + \frac{8}{5}$

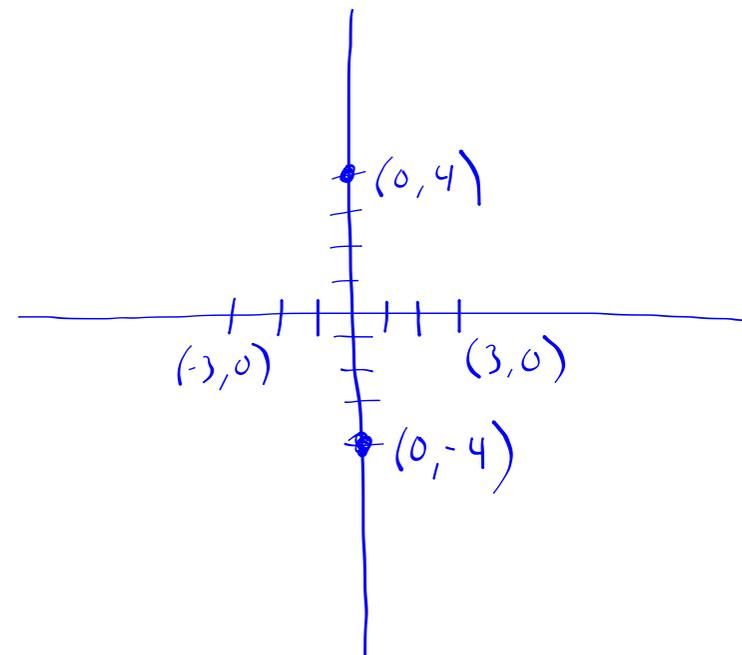10. Foci $(0,\pm4)$, length of conjugate axis 6

$$c^2 = a^2 + b^2$$

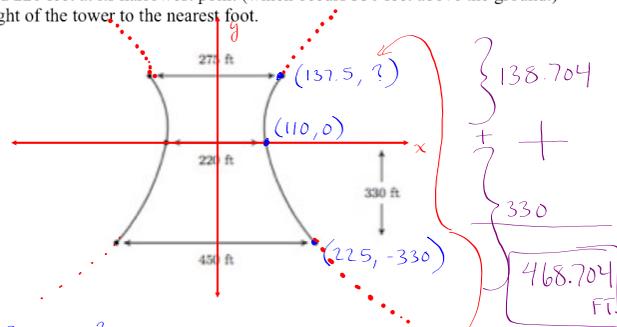$$c^2 = a^2 + 9$$

$$16 = a^2 + 9$$

$$-9 \qquad -9$$

$$7 = a^2$$

$$\frac{y^2}{7} - \frac{x^2}{9} = 1$$

14. 'Natural Draft' cooling towers are often shaped as **hyperboloids of revolution.** Each vertical cross section of these towers is a hyperbola. Suppose the tower is 450 feet wide at the base, 275 feet wide at the top, and 220 feet at its narrowest point (which occurs 330 feet above the ground.) Determine the height of the tower to the nearest foot.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{110^2} - \frac{y^2}{b^2} = 1$$

$$\frac{(225)^2}{(110)^2} - \frac{(-330)^2}{b^2} = 1$$

$$4.184 - \frac{108900}{b^2} = 1$$

$$(b^2)\frac{-108900}{b^2} = -3.184(b^2)$$

$$\frac{-108900}{-3.184} = \frac{-3.184b^2}{-3.184}$$

$$\sqrt{34202.3} = \sqrt{b^2}$$

$$b \approx 184.939$$

$$\frac{x^2}{110^2} - \frac{y^2}{184.939^2} = 1$$

$$\frac{(137.5)^2}{(110)^2} - \frac{y^2}{(184.939)^2} = 1$$

$$1.5625 - \frac{y^2}{34202.4} = 1$$

$$(34202.4)\frac{-y^2}{34202.4} = -0.5625(34202.4)$$

$$-y^2 = -19238.9$$

$$\sqrt{y^2} = \sqrt{19238.9}$$

$$y \approx 138.704$$

138.704
+
330
_____
468.704 Ft.