Notes 1.3 - Composite and Inverse Functions

Given $f(x)=2x-1$ and $g(x)=x^2-2x$

$(-5)^2-2(-5)$

$25+10$

find $g(f(-2))=35$

What would be the inverse of $f(x)$?

$f^{-1}(x)=\frac{x+1}{2}$

Notes 1.3 - Composite and Inverse Functions
* EXPLORATION *

What is a composite function?

What is an inverse?

Notes 1.3 - Composite and Inverse Functions

Given

$f(x) = 2x - 1$ $\quad g(x) = x^2 - 2x$ $\quad h(x) = 4x$

$g(h(0)) = 0$ $\quad h(f(-1)) = -12$

$f(g(x))$

$f(g(x)) = 2(x^2 - 2x) - 1$

$f(g(x)) = 2x^2 - 4x - 1$

$g(f(x))$

$g(f(x)) = (2x-1)^2 - 2(2x-1)$

$g(f(x)) = (2x-1)(2x-1) - 4x + 2$

$g(f(x)) = 4x^2 - 4x - 4x + 1 - 4x + 2$

$g(f(x)) = 4x^2 - 8x + 3$

Notes 1.3 - Composite and Inverse Functions

If $f(x)$ is a function, write the inverse as $f^{-1}(x)$.

To find the inverse of a function, simply switch x and y, and solve for y.

Notes 1.3 - Composite and Inverse Functions

2 ways to verify inverses

1) Graphing

- The two functions must reflect across the line $y = x$.

Verify that $f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x-1}$ are inverses of each other graphically.

Notes 1.3 - Composite and Inverse Functions

Use inverse operations to write the inverse of $g(x) = \frac{x+3}{4}$

$y = \frac{x+3}{4}$

$x = \frac{y+3}{4}$

$(4)x = \frac{y+3}{4}(4)$

$4x = y + 3$

$-3 \quad -3$

$y = 4x - 3$

$g^{-1}(x) = 4x - 3$

Notes 1.3 - Composite and Inverse Functions

Use inverse operations to write the inverse of $h(x) = \frac{4}{x+1} - 3$.

$y = \frac{4}{x+1} - 3$

$x = \frac{4}{y+1} - 3$

$+3 \qquad +3$

$(y+1)(x+3) = \frac{4}{y+1}(y+1)$

$\frac{(y+1)(x+3)}{x+3} = \frac{4}{x+3}$

$y + 1 = \frac{4}{x+3} - 1$

-1

$h^{-1}(x) = \frac{4}{x+3} - 1$

Notes 1.3 - Composite and Inverse Functions

Find the inverse of $h(x)=\frac{\sqrt[3]{x-1}}{2}$ and verify.

$y=\frac{\sqrt[3]{x-1}}{2}$

$2x=\frac{\sqrt[3]{y-1}}{2}$ (2)

$(2x)^3=\left(\sqrt[3]{y-1}\right)^3$

$8x^3=y-1$

$+1 \quad +1$

$h^{-1}(x)=8x^3+1$

Notes 1.3 - Composite and Inverse Functions

Find the inverse of $f(x)=x^2-4x+12$ and verify.

$y=x^2-4x+12$

$x=y^2-4y+12$
$-12 \quad -12$

$x-12=y^2-4y+4$
$+4$

$(y-2)(y-2)$

$x-8=(y-2)^2$

$\sqrt{x-8}=\sqrt{(y-2)^2}$

$\pm\sqrt{x-8}=y-2$
$+2 \quad +2$

$y=\pm\sqrt{x-8}+2$

$f^{-1}(x)=2\pm\sqrt{x-8}$

* COMPLETING THE SQUARE
ax^2+bx+c

Notes 1.3 - Composite and Inverse Functions

Find the inverse of $f(x)=3x^2-18x+2$ and verify.

$y=3x^2-18x+2$

$x=3y^2-18y+2$

$-2 \qquad -2$

$x-2=3y^2-18y$

$x-2=3(y^2-6y+9)$

$+27$

$\frac{x+25}{3}=\frac{3(y-3)^2}{3}$

$\sqrt{\frac{x+25}{3}}=\sqrt{(y-3)^2}$

$\pm\sqrt{\frac{x+25}{3}}=y-3$

$+3 \qquad +3$

$f^{-1}(x)=3\pm\sqrt{\frac{x+25}{3}}$