



Notes 1.4 - Difference Quotient

* WARM UP *

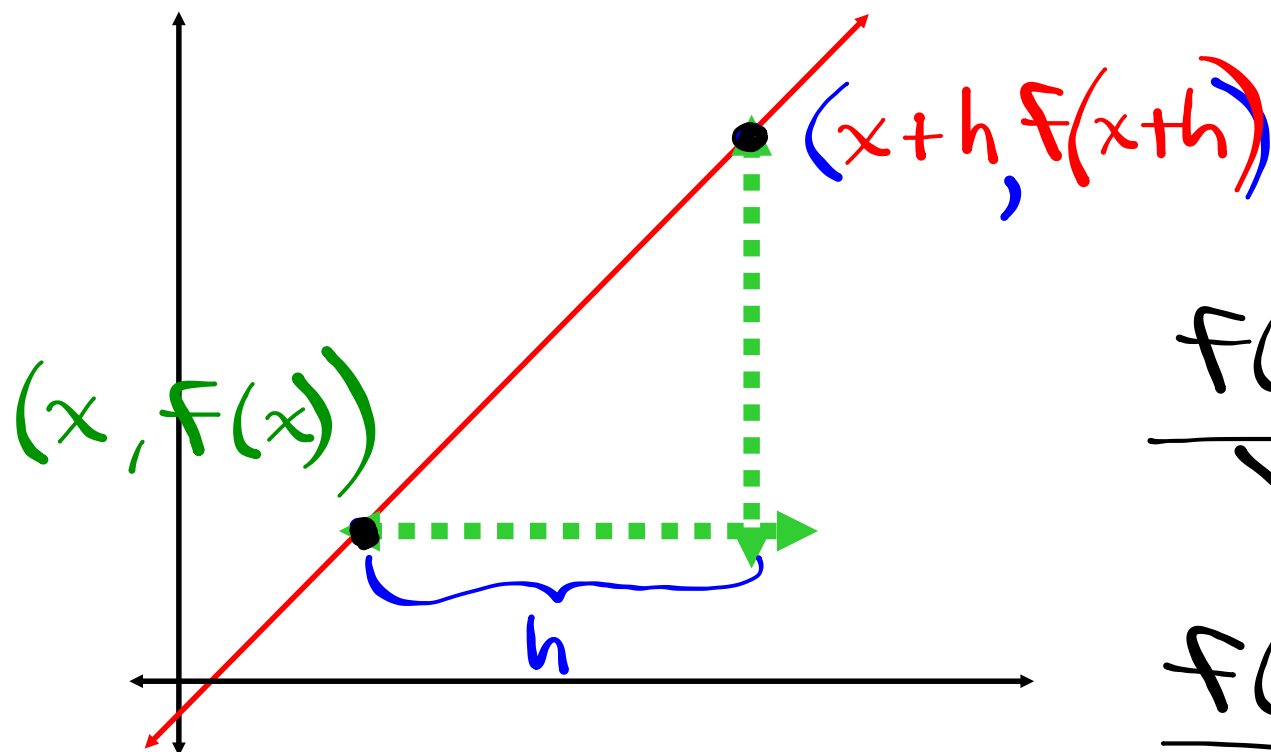
Find the rate of change of a line that contains the points $(-2, 0)$ and $(3, 5)$.

$$m = \frac{5}{5} \quad m = 1$$



Notes 1.4 - Difference Quotient

* EXPLORATION *



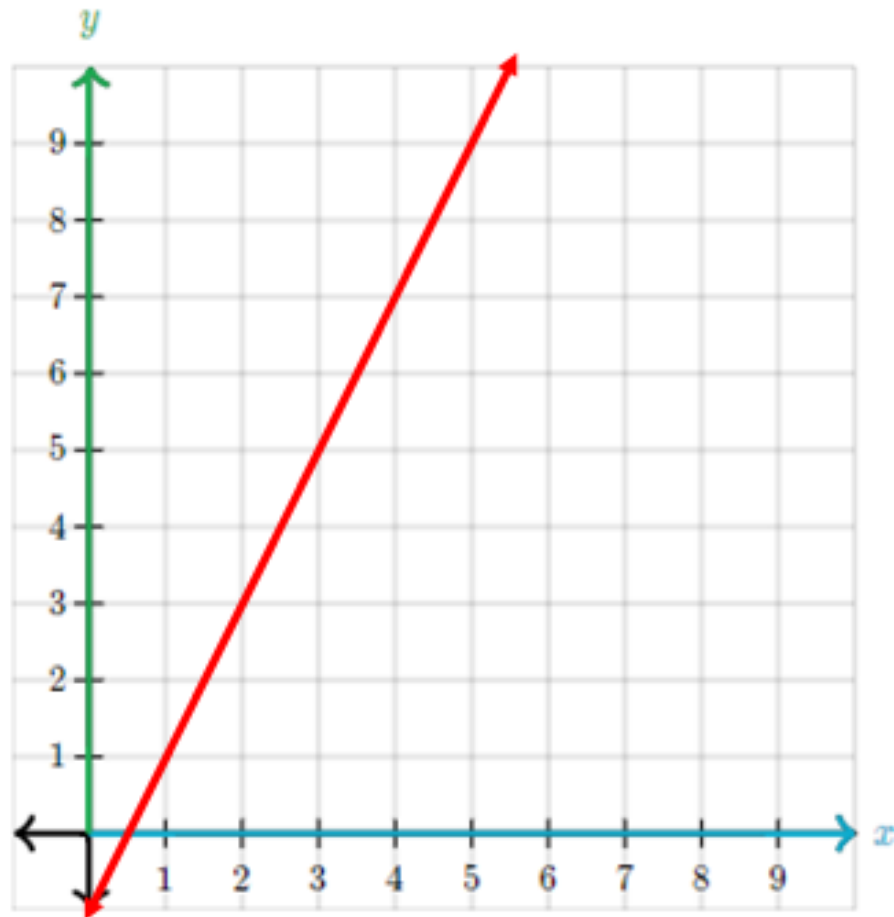
$$\frac{f(x+h) - f(x)}{\cancel{x+h} - \cancel{x}}$$
$$\frac{f(x+h) - f(x)}{h}$$





Notes 1.4 - Difference Quotient

* EXPLORATION *



$$f(x) = 2x - 1$$





Notes 1.4 - Difference Quotient

Definition of the difference quotient

$$\frac{f(x+h) - f(x)}{h}$$





Notes 1.4 - Difference Quotient

$$\frac{f(x+h) - f(x)}{h}$$

Given $f(x) = 2x - 5$, find the difference quotient.

$$f(x+h) = 2(x+h) - 5 = 2x + 2h - 5$$

$$f(x) = 2x - 5$$

$$\frac{2x + 2h - 5 - (2x - 5)}{h}$$

$$\frac{\cancel{2x} + 2h - \cancel{5} - \cancel{2x} + \cancel{5}}{h} = \frac{2h}{h}$$

$$\boxed{2}$$

$$f(x) = 3x + 2$$





Notes 1.4 - Difference Quotient

$$\frac{f(x+h) - f(x)}{h}$$

Given $f(x) = 3 - x$, find the difference quotient.

$$\frac{3 - (x+h) - (3 - x)}{h}$$

$$\frac{\cancel{3} - \cancel{x} - h - \cancel{3} + \cancel{x}}{h} = -\frac{h}{h} = -1$$





Notes 1.4 - Difference Quotient

$$\frac{f(x+h) - f(x)}{h}$$

Given $f(x) = x^2$, find the difference quotient.

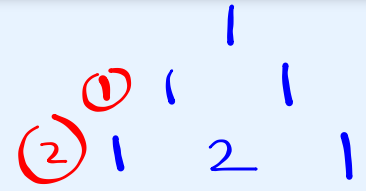
$$\frac{(x+h)^2 - x^2}{h}$$
$$\frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$

$$(x+h)^2$$
$$= (x+h)(x+h)$$
$$x^2 + 2xh + h^2$$

$$\frac{2xh + h^2}{h} = \frac{\cancel{h}(2x+h)}{\cancel{h}}$$



$$2x+h$$



$$\cancel{1}x^2 + 2xh + 1h^2$$
$$2x+h$$



Notes 1.4 - Difference Quotient

$$\frac{f(x+h) - f(x)}{h}$$

Given $f(x) = 2x^2 - x$, find the difference quotient.

$$\frac{2(x+h)^2 - (x+h) - (2x^2 - x)}{h}$$
$$\frac{2(x^2 + 2xh + h^2) - x - h - 2x^2 + x}{h}$$

$$\frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{x} - h - \cancel{2x^2} + \cancel{x}}{h}$$

$$\frac{4xh + 2h^2 - h}{h}$$

$$\frac{\cancel{h}(4x + 2h - 1)}{\cancel{h}}$$

$$4x + 2h - 1$$





Notes 1.4 - Difference Quotient

$$\frac{f(x+h) - f(x)}{h}$$

Given $f(x) = \frac{1}{x+3}$, find the difference quotient.

$$\frac{\frac{1}{x+3} - \frac{1}{x+h+3}}{h}$$

$$\frac{\cancel{x+3} - \cancel{x} - h - \cancel{3}}{(x+3)(x+h+3)} \cdot \frac{1}{h}$$

$$\frac{-h}{(x+3)(x+h+3)} \cdot \frac{1}{h}$$

$$\frac{-1}{(x+3)(x+h+3)}$$





Notes 1.4 - Difference Quotient

$$\frac{f(x+h) - f(x)}{h}$$

Given $f(x) = \sqrt{x-5}$, find the difference quotient.

$$\frac{(\sqrt{x+h-5} - \sqrt{x-5})}{h} \cdot \frac{(\sqrt{x+h-5} + \sqrt{x-5})}{(\sqrt{x+h-5} + \sqrt{x-5})}$$

$$\frac{\cancel{x+h-5} - \cancel{(x-5)}}{h(\sqrt{x+h-5} + \sqrt{x-5})}$$

$$\frac{\cancel{h}}{h(\sqrt{x+h-5} + \sqrt{x-5})}$$

$$\frac{1}{\sqrt{x+h-5} + \sqrt{x-5}}$$



$$\frac{(\sqrt{x+3})(\sqrt{x-3})}{x-9}$$

$$\frac{(x+2)(x-2)}{x^2-4}$$

$$f(x) = x^4$$

$$\frac{(x+h)^4 - x^4}{h}$$

$$\frac{(x+h)(x+h)(x+h)(x+h) - x^4}{h}$$

$$\frac{(x+h)^2(x+h)^2 - x^4}{h}$$

$$\frac{(x^2 + 2xh + h^2)(x^2 + 2xh + h^2) - x^4}{h}$$

$$\frac{\cancel{x^4} + 2x^3h + x^2h^2 + 2x^3h + 4x^2h^2 + 2xh^3 + x^2h^2 + 2xh^3 + h^4 - \cancel{x^4}}{h}$$

$$\frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}$$

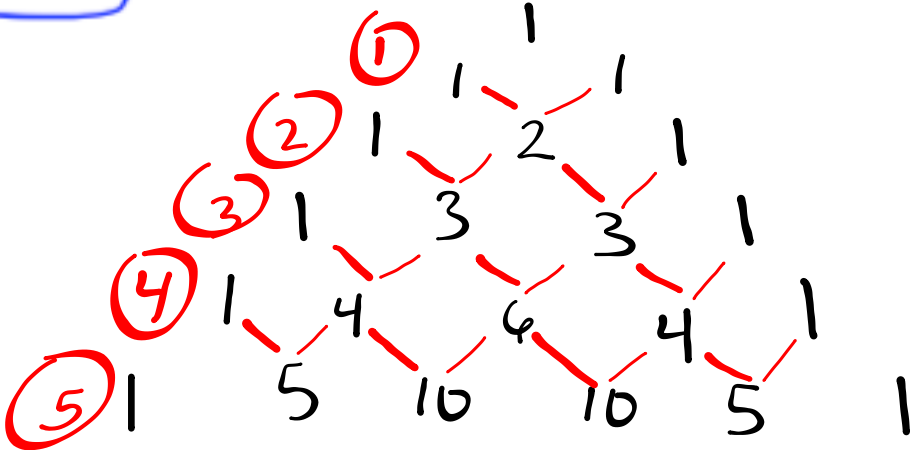
$$\frac{h(4x^3 + 6x^2h + 4xh^2 + h^3)}{h}$$

$$\boxed{4x^3 + 6x^2h + 4xh^2 + h^3}$$

$$\frac{f(x+h) - f(x)}{h}$$

$$4x^3 + 6x^2h + 4xh^2 + h^3$$

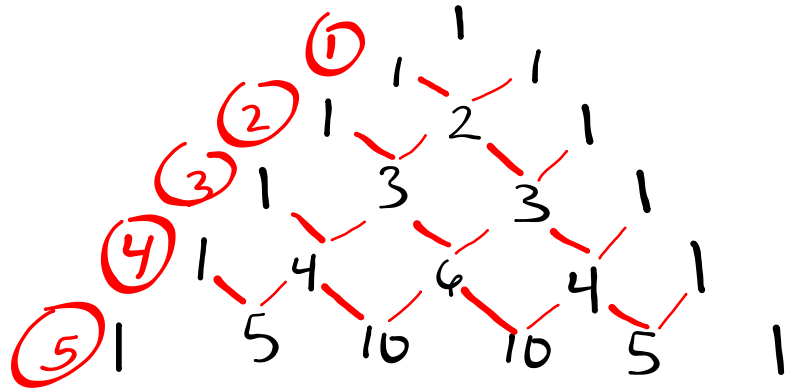
$$f(x) = x^4$$



$$\cancel{1}x^4 + \cancel{4}x^3\cancel{h} + \cancel{6}x^2\cancel{h}^2 + \cancel{4}x\cancel{h}^3 + \cancel{1}h^4$$

$$4x^3 + 6x^2h + 4xh^2 + h^3$$

$$f(x) = 2x^3 - 5x^5$$



~~$$1x^3 + 3x^2h + 3xh^2 + 1h^3$$~~

~~$$1x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + 1h^5$$~~

$$2(3x^2 + 3xh + h^2) - 5(5x^4 + 10x^3h + 10x^2h^2 + 5xh^3 + h^4)$$

$$6x^2 + 6xh + 2h^2 - 25x^4 - 50x^3h - 50x^2h^2 - 25xh^3 - 5h^4$$