Notes 2.3 - Properties of Logarithms

* WARM UP *

Evaluate.

$\log_7 49 = 2$

$\log_7 \left(\frac{1}{49}\right) = -2$

$\log_{49} 7 = \frac{1}{2}$

$\log_{49} \left(\frac{1}{7}\right) = -\frac{1}{2}$

$\ln e^{49} = 49$

$\log_7 7 = 1$

Notes 2.3 - Properties of Logs

* EXPLORATION *

Recall the product property of exponents.

$$b^m b^n = b^{m+n}$$

Let's prove the product property of logarithms.

Let $x = b^m$ and $y = b^n$

Notes 2.3 - Properties of Logs

Remember that to *multiply* powers with the same base, you *add* exponents.

$$b^m b^n = b^{m+n}$$

Product Property of Logarithms

For any positive numbers m, n, and $b (b \neq 1)$,

WORDS	NUMBERS	ALGEBRA
The logarithm of a product is equal to the sum of the logarithms of its factors.	$\log_3 1000 = \log_3 (10 \cdot 100)$ $= \log_3 10 + \log_3 100$	$\log_b mn = \log_b m + \log_b n$

Notes 2.3 - Properties of Logs

Remember that to *divide* powers with the same base, you *subtract* exponents

$$\frac{b^m}{b^n} = b^{m-n}$$

Quotient Property of Logarithms

For any positive numbers m, n, and $b\,(b \neq 1)$,

WORDS	NUMBERS	ALGEBRA
The logarithm of a quotient is the logarithm of the dividend minus the logarithm of the divisor.	$\log_5\left(\frac{16}{2}\right) = \log_5 16 - \log_5 2$	$\log_b \frac{m}{n} = \log_b m - \log_b n$

Notes 2.3 - Properties of Logs

$(b^m)^n = b^{m \cdot n}$

Power Property of Logarithms

For any real number p and positive numbers a and $b\,(b \neq 1)$,

WORDS	NUMBERS	ALGEBRA
The logarithm of a power is the product of the exponent and the logarithm of the base.	$\log 10^3$ $\log(10 \cdot 10 \cdot 10)$ $\log 10 + \log 10 + \log 10$ $3 \log 10$	$\log_b a^p = p \log_b a$

Notes 2.3 - Properties of Logs

Condense as a single logarithm and simplify, if possible.

$\log_6 4 + \log_6 9$ $= \log_6 36 = 2$

$\log_{\frac{1}{3}} 27 + \log_{\frac{1}{3}} \frac{1}{9}$ $\log_{\frac{1}{3}} 3 = -1$

Notes 2.3 - Properties of Logs

Condense as a single logarithm and simplify, if possible.

$\log_5 100 - \log_5 4$ $= \log_5 25 = 2$

$\log_7 49 - \log_7 7$ $= \log_7 7 = 1$

Notes 2.3 - Properties of Logs

Express as a product and simplify, if possible.

$\log_2 32^6 = 6 \log_2 32 = 6 \cdot 5 = 30$

$\log_{16} 4^{20} = 20 \log_{16} 4 = 10$

$\log_5 25^2 = 2 \log_5 25 = 4$

Notes 2.3 - Properties of Logs

Expand.

$$\log_6 \frac{x^4}{y}$$

$\log_6 x^4 - \log_6 y$

$\boxed{4\log_6 x - \log_6 y}$

$$\ln\left(w\sqrt[3]{u \cdot v}\right)$$

$\ln(w(uv)^{1/3})$

$\ln(w u^{1/3} \cdot v^{1/3})$

$\ln w + \ln u^{1/3} + \ln v^{1/3}$

$\boxed{\ln w + \frac{\ln u}{3} + \frac{\ln v}{3}}$

Notes 2.3 - Properties of Logs

Condense.

$5\log_8 x + 5\log_8 y$

$\log_8(x^5 \cdot y^5)$

$\log_8 (xy)^5$

$\ln z + \frac{\ln x}{3} + \frac{\ln y}{3}$

$\ln\left(z \cdot x^{\frac{1}{3}} y^{\frac{1}{3}}\right)$

$\ln\left(z\sqrt[3]{xy}\right)$