Notes 2.4 - Solving Exponential and Logarithmic Equations

* WARM UP *

Solve for x.

$$\log_3 x = 2$$

$x = 9$

Which domain values are not allowed in the expression below?

$$\log_3 x$$

$x \leq 0$

Notes 2.4 - Solving Exponential and Logarithmic Equations

What is an exponential equation?

$$3^{x+2} = 15$$

Notes 2.4 - Solving Exponential and Logarithmic Equations

There are 2 ways to solve exponential equations.

1] Write both sides of the equation in terms of the same base.

*** Keep answers exact, no calculator necessary. ***

Notes 2.4 - Solving Exponential and Logarithmic Equations

Solve and check.

$9^{8-x} = 27^{x-3}$

$(3^2)^{8-x} = (3^3)^{x-3}$

$3^{16-2x} = 3^{3x-9}$

$16-\cancel{2x} = 3x-9$

$+9 \; +\cancel{2x} \quad +2x \; +9$

$\frac{25}{5} = \frac{5x}{5}$

$\boxed{x=5}$

$3^{2x} = 27$

$3^{2x} = 3^3$

$\frac{\cancel{2}x}{\cancel{2}} = \frac{3}{2}$

$\boxed{x = \frac{3}{2}}$

Notes 2.4 - Solving Exponential and Logarithmic Equations

There are 2 ways to solve exponential equations.

2] Take the logarithm of both sides.

Recall... $\boxed{\log_b b^x = x}$

*** Keep answers exact, no calculator necessary. ***

Notes 2.4 - Solving Exponential and Logarithmic Equations

Solve and check.

$4^{x-1} = 5$

$4^{x-1} = 5$

$\cancel{\log_4} 4^{x-1} = \log_4 5$

$x \cancel{-1} = \log_4 5 + 1$
$\cancel{+1}$

$\boxed{x = 1 + \log_4 5}$

$7^{-x} = 21$

$\cancel{\log_7} 7^{-x} = \log_7 21$

$\frac{-x}{-1} = \frac{\log_7 21}{-1}$

$\boxed{x = -\log_7 21}$

Notes 2.4 - Solving Exponential and Logarithmic Equations

What is a logarithmic equation?

$$\log_2(x+4) = 2$$

Notes 2.4 - Solving Exponential and Logarithmic Equations

We can apply the same base exponent rule to logarithms.

$$\boxed{\text{If } b^x = b^y \text{, then } x = y.}$$

Therefore, we can also say...

$$\boxed{\text{If } \log_b x = \log_b y \text{ , then } x = y \text{ .}}$$

$$\log_3(2x-1) = \log_3(x+5)$$

$$2x\cancel{-1} = \cancel{x}+5$$
$$-x \quad \cancel{+1} \quad -\cancel{x} \quad +1$$
$$\boxed{x=6}$$

Notes 2.4 - Solving Exponential and Logarithmic Equations

To solve a logarithmic equation, recall...

$$\boxed{b^{\log_b x} = x}$$

What logarithmic values result in errors?

$$\log_b x,\ x > 0$$

Notes 2.4 - Solving Exponential and Logarithmic Equations

Solve and check.

$$\log_6(2x-1) = -1$$

$$6^{\cancel{\log_6}(2x-1)} = 6^{-1}$$

$$2x \cancel{-1} = \frac{1}{6} + \frac{1(6)}{1(6)}$$
$$\quad\; +1$$

$$\frac{\cancel{2}x}{\cancel{2}} = \frac{7}{6} \cdot \frac{1}{2}$$

$$\boxed{x = \frac{7}{12}}$$

$$\log_4 100 - \log_4(x+1) = 1$$

$$\log_4\left(\frac{100}{x+1}\right) = 1$$

$$4^{\log_4\left(\frac{100}{x+1}\right)} = 4^1$$

$$\cancel{(x+1)}\frac{100}{\cancel{x+1}} = 4(x+1)$$

$$\frac{100}{4} = \frac{\cancel{4}(x+1)}{\cancel{4}}$$

$$25 = x+1$$
$$-1 \qquad -1$$

$$\boxed{x = 24}$$

Notes 2.4 - Solving Exponential and Logarithmic Equations

Solve and check.

$$\log_5 x^4 = 8$$

$$\frac{\cancel{4}\log_5 x}{\cancel{4}} = \frac{8}{4}$$

$$\log_5 x = 2$$

$$\cancel{5}^{\log_5 x} = 5^2$$

$$\boxed{x = 25}$$

$$\log_{12} x + \log_{12}(x+1) = 1$$

$$\log_{12}(x(x+1)) = 1$$

$$\log_{12}(x^2+x) = 1$$

$$\cancel{12}^{\log_{12}(x^2+x)} = 12^1$$

$$x^2 + x = 12$$
$$-12 \quad -12$$

$$x^2 + x - 12 = 0$$

$$(x+4)(x-3) = 0$$

$$\cancel{x = -4}$$ EXTRANEOUS SOLUTION

$$\boxed{x = 3}$$

Notes 2.4 - Solving Exponential and Logarithmic Equations

Solve and check.

$$3 = \log 8 + 3\log x$$

$3 = \log 8 + \log x^3$

$3 = \log 8x^3$

$10^3 = 10^{\log 8x^3}$

$\frac{1000}{8} = \frac{8x^3}{8}$

$\sqrt[3]{\frac{1000}{8}} = \sqrt[3]{x^3}$

$\frac{10}{2} = x$

$\boxed{x = 5}$

$$2\log x - \log 4 = 0$$

$\log x^2 - \log 4 = 0$

$\log \frac{x^2}{4} = 0$

$\cancel{10}^{\log \frac{x^2}{4}} = 10^0$

$\cancel{4} \cdot \frac{x^2}{4} = 1(4)$

$\sqrt{x^2} = \sqrt{4}$

$x = \pm 2$

$\boxed{x = 2}$

$\log_5 x^2$

$\log_5 x^2$ $\log_5 (x+3)$

$\log_5 5^2$ $\log_5 -5^2$

$x > 0$