



Notes 2.4 - Solving Exponential and Logarithmic Equations

* WARM UP *

Solve for x.

$$\log_3 x = 2$$

$$x = 9$$

Which domain values are not allowed in the expression below?

$$\log_3 x$$

$$x \leq 0$$





Notes 2.4 - Solving Exponential and Logarithmic Equations

What is an exponential equation?

$$3^{x+2} = 15$$





Notes 2.4 - Solving Exponential and Logarithmic Equations

There are 2 ways to solve exponential equations.

1] Write both sides of the equation in terms of the same base.

*** Keep answers exact, no calculator necessary. ***





Notes 2.4 - Solving Exponential and Logarithmic Equations

Solve and check.

$$9^{8-x} = 27^{x-3}$$

$$(3^2)^{8-x} = (3^3)^{x-3}$$

$$3^{16-2x} = 3^{3x-9}$$

$$\begin{aligned} 16 - 2x &= 3x - 9 \\ +9 + 2x &+ 2x + 9 \end{aligned}$$

$$\frac{25}{5} = \frac{5x}{5}$$

$$\boxed{x=5}$$

$$3^{2x} = 27$$

$$3^{2x} = 3^3$$

$$\frac{2x}{2} = \frac{3}{2}$$

$$\boxed{x = \frac{3}{2}}$$





Notes 2.4 - Solving Exponential and Logarithmic Equations

There are 2 ways to solve exponential equations.

2] Take the logarithm of both sides.

Recall...

$$\log_b b^x = x$$

*** Keep answers exact, no calculator necessary. ***





Notes 2.4 - Solving Exponential and Logarithmic Equations

Solve and check.

$$4^{x-1} = 5$$

$$4^{x-1} = 5$$

$$\cancel{\log_4 4^{x-1}} = \log_4 5$$

$$\cancel{x-1} = \log_4 5 + 1$$

$$x = 1 + \log_4 5$$

$$7^{-x} = 21$$

$$\cancel{\log_7 7^{-x}} = \log_7 21$$

$$\frac{-x}{-1} = \frac{\log_7 21}{-1}$$

$$x = -\log_7 21$$





Notes 2.4 - Solving Exponential and Logarithmic Equations

What is a logarithmic equation?

$$\log_2(x+4) = 2$$





Notes 2.4 - Solving Exponential and Logarithmic Equations

We can apply the same base exponent rule to logarithms.

$$\text{If } b^x = b^y, \text{ then } x = y.$$

Therefore, we can also say...

$$\text{If } \log_b x = \log_b y, \text{ then } x = y.$$



$$\log_3(2x-1) = \log_3(x+5)$$

$$\begin{array}{r} 2x+1 = x+5 \\ -x \quad -x \quad +1 \quad +1 \\ \hline \end{array}$$

$$x=6$$



Notes 2.4 - Solving Exponential and Logarithmic Equations

To solve a logarithmic equation, recall...

$$b^{\log_b x} = x$$

What logarithmic values result in errors?

$$\log_b x, x > 0$$





Notes 2.4 - Solving Exponential and Logarithmic Equations

Solve and check.

$$\log_6(2x-1) = -1$$

$$\cancel{6}^{\log_6(2x-1)} = 6^{-1}$$

$$2x-1 = \frac{1}{6} + \frac{1(6)}{1(6)}$$

$$\cancel{\frac{2x}{2}} = \frac{7}{6} \cdot \frac{1}{2}$$

$$\boxed{x = \frac{7}{12}}$$

$$\log_4 100 - \log_4(x+1) = 1$$

$$\log_4\left(\frac{100}{x+1}\right) = 1$$

$$\cancel{4}^{\log_4\left(\frac{100}{x+1}\right)} = 4^1$$

$$\cancel{\frac{(x+1)100}{x+1}} = 4(x+1)$$

$$\frac{100}{4} = \cancel{\frac{4(x+1)}{4}}$$

$$25 = x+1$$
$$-1 \quad -1$$

$$\boxed{x = 24}$$





Notes 2.4 - Solving Exponential and Logarithmic Equations

Solve and check.

$$\log_5 x^4 = 8$$

~~$$\frac{\log_5 x^4}{4} = \frac{8}{4}$$~~

$$\log_5 x = 2$$

$$5^{\log_5 x} = 5^2$$

$$\boxed{x = 25}$$

$$\log_{12} x + \log_{12} (x+1) = 1$$

$$\log_{12} (x(x+1)) = 1$$

$$\log_{12} (x^2 + x) = 1$$

~~$$12^{\log_{12} (x^2 + x)} = 12^1$$~~

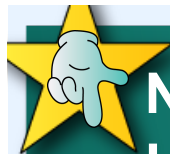
$$\begin{aligned} x^2 + x &= 12 \\ -12 \quad -12 \end{aligned}$$

$$x^2 + x - 12 = 0$$

$$(x+4)(x-3) = 0$$

~~$$x = -4$$~~ EXTRANEOUS SOLUTION
$$\boxed{x = 3}$$





Notes 2.4 - Solving Exponential and Logarithmic Equations

Solve and check.

$$3 = \log 8 + 3 \log x$$

$$3 = \log 8 + \log x^3$$

$$3 = \log 8x^3$$

$$10^3 = 10^{\log 8x^3}$$

$$\frac{1000}{8} = \frac{8x^3}{8}$$

$$\sqrt[3]{\frac{1000}{8}} = \sqrt[3]{x^3}$$

$$\frac{10}{2} = x$$

$$\boxed{x=5}$$

$$2 \log x - \log 4 = 0$$

$$\log x^2 - \log 4 = 0$$

$$\log \frac{x^2}{4} = 0$$

$$\cancel{10^{\log \frac{x^2}{4}}} = 10^0$$

$$\cancel{\frac{x^2}{4}} = (4)$$

$$\sqrt{x^2} = \sqrt{4}$$

$$x = \pm 2$$

$$\boxed{x=2}$$

$$\log_5 x^2$$



$$\log_5 x^2$$

$$\log_5(x+3)$$

$$\log_5 5^2$$

$$\log_5^{-} 5^2$$

$$x > 0$$

