Notes 3.3 - More Trig Equations and Trig Identities Intro

* WARM UP *

Evaluate.

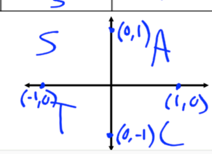
$$\sin\frac{5\pi}{6} = \frac{1}{2}$$

Solve $0 \le \theta < 2\pi$

$$-2+\sec^2\theta = -\cancel{\sec\theta}$$

$+\sec\theta \qquad +\sec\theta$

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	UND



$$\sec^2\theta + \sec\theta - 2 = 0$$

$$(\sec\theta - 1)(\sec\theta + 2) = 0$$

$\sec\theta = 1$

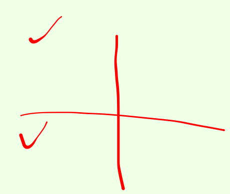
$\cos\theta = 1$

$\boxed{\theta = 0}$

$\sec\theta = -2$

$\cos\theta = -\frac{1}{2}$

$\boxed{\theta = \frac{2\pi}{3}, \frac{4\pi}{3}}$



Notes 3.3 - More Trig Equations and Trig Identities Intro

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Notes 3.3 - More Trig Equations and Trig Identities Intro

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

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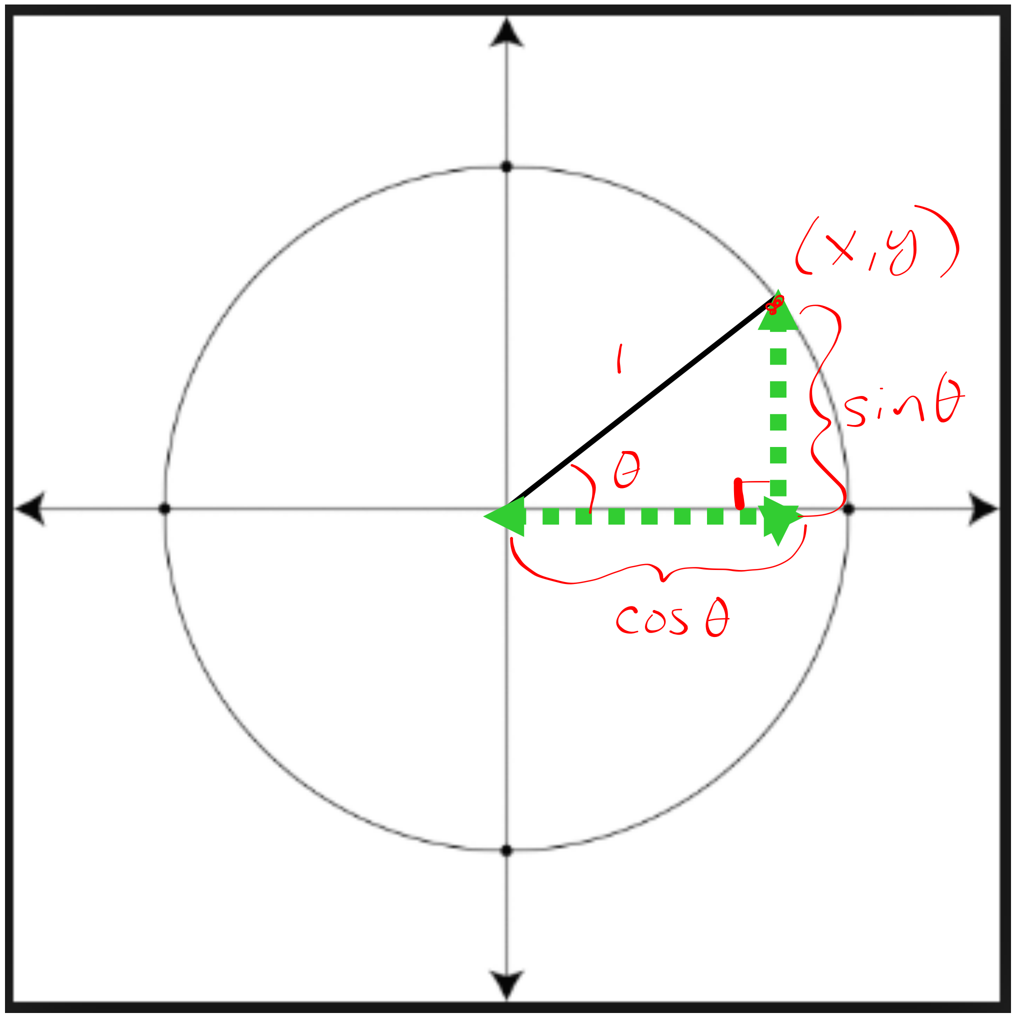
* EXPLORATION *

Proof



$$\sin^2\theta + \cos^2\theta = 1$$

Notes 3.3 - More Trig Equations and Trig Identities Intro

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $\quad$ $1 + \tan^2\theta = \sec^2\theta$ $\quad$ $1 + \cot^2\theta = \csc^2\theta$

Notes 3.3 - More Trig Equations and Trig Identities Intro

Solve each equation for $0 \le \theta < 2\pi$.

$$\sin^2\theta = \cos^2\theta - \sin\theta$$

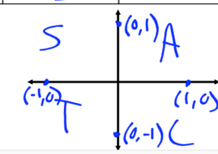
$\sin^2\theta = (1-\sin^2\theta) - \sin\theta$

$\sin^2\theta = 1-\sin^2\theta - \sin\theta$

$2\sin^2\theta + \sin\theta - 1 = 0$

$(2\sin\theta - 1)(\sin\theta + 1) = 0$

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	UND



$\frac{2\sin\theta}{2} = \frac{1}{2}$

$\sin\theta = \frac{1}{2}$

$\boxed{\theta = \frac{\pi}{6}, \frac{5\pi}{6}}$

$\sin\theta = -1$

$\boxed{\theta = \frac{3\pi}{2}}$

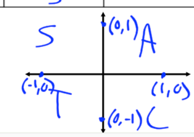
Notes 3.3 - More Trig Equations and Trig Identities Intro

Solve each equation for $0 \le \theta < 2\pi$.

$\sin\theta + 2\cos\theta = 3\cos\theta + 1$

$-2\cos\theta \quad -2\cos\theta$

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	UND



$(\sin\theta)^2 = (\cos\theta + 1)^2 \qquad (\cos\theta + 1)(\cos\theta + 1)$

$\sin^2\theta = \cos^2\theta + 2\cos\theta + 1$

$(1 - \cos^2\theta) = \cos^2\theta + 2\cos\theta + 1$

$\cancel{1} - \cos^2\theta = \cos^2\theta + 2\cos\theta + \cancel{1}$

$-\cancel{1} + \cos^2\theta \qquad + \cos^2\theta \qquad -\cancel{1}$

$2\cos^2\theta + 2\cos\theta = 0$

$2\cos\theta(\cos\theta + 1) = 0$

$\frac{2\cos\theta}{2} = \frac{0}{2} \qquad \cos\theta = -1$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$ ✓, $\cancel{\frac{3\pi}{2}} \qquad \theta = \pi$ ✓

$\sin\left(\frac{\pi}{2}\right) + 2\cos\left(\frac{\pi}{2}\right) = 3\cos\left(\frac{\pi}{2}\right) + 1$

$1 + 2(0) = 3(0) + 1$

$1 = 1$ ✓

$\sin\left(\frac{3\pi}{2}\right) + 2\cos\left(\frac{3\pi}{2}\right) = 3\cos\left(\frac{3\pi}{2}\right) + 1$

$-1 + 2(0) = 3(0) + 1$

$-1 = 1$

$\sin(\pi) + 2\cos(\pi) = 3\cos(\pi) + 1$

$\cancel{0} + 2(-1) = 3(-1) + 1$

$-2 = -2$

Notes 3.3 - More Trig Equations and Trig Identities Intro

Tip: We are not solving anything so stick to one side of the equation. We are verifying identities that are already proved to be true.

Notes 3.3 - More Trig Equations and Trig Identities Intro

Verify the identity.

$$\boxed{\frac{\cos x}{\csc x}} = \frac{\sin x}{\sec x}$$

$$\frac{1}{\sec x} \cdot \frac{1}{\csc x}$$

$$\frac{1}{\sec x} \cdot \frac{\sin x}{1}$$

$$\frac{\sin x}{\sec x} \quad \therefore$$

Notes 3.3 - More Trig Equations and Trig Identities Intro

Verify the identity.

$$\frac{\sec^2 x}{\sin x} = \frac{\csc x}{\cos^2 x}$$

$$\frac{1}{\cos^2 x} \cdot \frac{1}{\sin x}$$

$$\frac{1}{\cos^2 x} \cdot \frac{\csc x}{1}$$

$$\frac{\csc x}{\cos^2 x}$$

Notes 3.3 - More Trig Equations and Trig Identities Intro

Verify the identity.

$$\boxed{\frac{\sin x - \tan x}{\tan x}} = \cos x - 1$$

$$\frac{\sin x}{\tan x} - 1$$

$$\frac{\sin x}{\frac{\sin x}{\cos x}} - 1$$

$$\frac{\cancel{\sin x}}{1} \cdot \frac{\cos x}{\cancel{\sin x}} - 1$$

$\cos x - 1$ ∴

Notes 3.3 - More Trig Equations and Trig Identities Intro

Verify the identity.

$$\boxed{\csc^2 x + \tan^2 x} = \sec^2 x + \cot^2 x$$

$$(1 + \cot^2 x) + (\sec^2 x - 1)$$

$$\cancel{1} + \cot^2 x + \sec^2 x - \cancel{1}$$

$$\sec^2 x + \cot^2 x \quad \therefore$$