Unit 2: Functions and Their Graphs

* UNPACKING EA 1 *

Skill	Topic
find	quartic functions, ~~y-intercepts~~
graph, sketch	quartic model, ~~polynomial~~
explain	quartic model, x~~-intercepts (zeros),~~ complex zeros, ~~turning points~~
factor	polynomials
write	~~functions,~~ ~~polynomial~~ (~~standard~~)
create	inequality

Lesson 9-1: Data and Polynomial Models

Learning Target Unpacking

Skill	Topic
compare	models
predict	regression

Lesson 9-2: Polynomial Functions

Learning Target Unpacking

Skill	Topic
Describe	
Analyze	Polynomial Functions
Graph	

Polynomials
Sunspots
Lesson 9-1 Data and Polynomial Models

ACTIVITY 9

Learning Targets:
- Compare models to best fit a data set.
- Use a polynomial regression to make predictions.

SUGGESTED LEARNING STRATEGIES: Summarizing, Paraphrasing, Look for a Pattern, Create Representations, Discussion Groups, Quickwrite

Dark areas called *sunspots* appear on the surface of the Sun. Sunspots last from a few days to a few weeks. Scientists who study sunspots have found that there appears to be a relationship over time between the number of sunspots and the year in which they occur.

The following data represent the number of sunspots for 1991 through 2000. Data were obtained from the National Geophysical Data Center.

Year	Years Since 1990	Sunspots
1991	1	146
1992	2	94
1993	3	55
1994	4	30
1995	5	18
1996	6	9
1997	7	22
1998	8	64
1999	9	93
2000	10	120

1. **Model with mathematics.** Fill in the table. Explain why a linear function would not be an appropriate model to represent the number of sunspots given the number of years since 1990.

• THE DIFFERENCES IN THE INPUTS/OUTPUTS ARE NOT CONSTANT

2. Would an exponential function be a good model for the data? Explain your reasoning.

• NO, THE NUMBER OF SUNSPOTS DECREASES THEN INCREASES.

My Notes

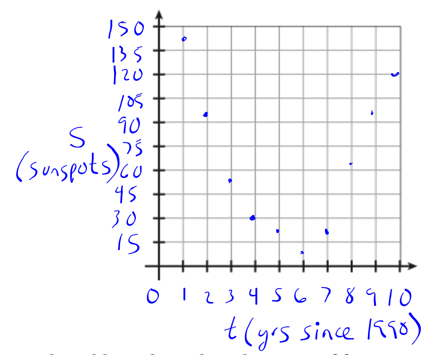
CONNECT TO SCIENCE

Sunspots appear in pairs. They are intense magnetic fields that break through the surface of the sun. The field lines leave through one sunspot and re-enter through another.

Sunspots were observed as early as 200 B.C. by the Chinese. It was not until the mid-nineteenth century that Rudolf Wolf devised a method for estimating daily solar activity.

My Notes

3. Graph the data from the table for the number of sunspots as a function of the number of years since 1990. Label both axes on the graph with an appropriate scale.



4. Examine the table and graph. What type of function could be used to model the data? Explain your reasoning.

- QUADRATIC, THE GRAPH FORMS THE GENERAL SHAPE OF A PARABOLA.

5. Use the regression capabilities of your graphing calculator. Find a model that best represents the data.

- $F(x) = -6.01x^2 - 67.51x + 204.98$

6. List the important features of the graph of this function.

minimum @ (5.62, 15.4)

y-int @ (0, 204.98)

NO ZEROS

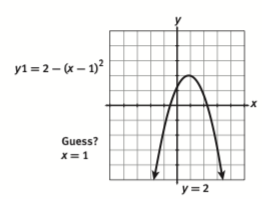
7. According to the model, in which year are the sunspot occurrences at a minimum?

$t = 5.62$

$\boxed{\text{SO BETWEEN 1995 AND 1996}}$

TECHNOLOGY TIP

A graphing calculator can be used to find the maximum or the minimum values of a quadratic function.



MATH TIP

The quadratic formula is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Lesson 9-1
Data and Polynomial Models

8. Use the model you found in Item 5 to determine the year(s) in which the number of sunspots is 200.

$200 = 6.011x^2 - 67.513x + 204.983$

$x = 0.07 \quad x = 11.2$

$\boxed{\text{1990 AND 2001}}$

9. Reason quantitatively. Do you think the quadratic model is accurate for the distant past and future? Explain.

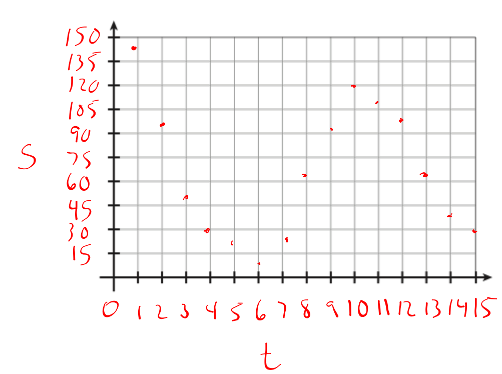
NO, THE DATA WILL ONLY WORK FOR AN IMMEDIATE TIME FRAME.

The table below shows sunspot data from the year 1991 to 2005.

Year	Sunspots
1991	146
1992	94
1993	55
1994	30
1995	18
1996	9
1997	22
1998	64

Year	Sunspots
1999	93
2000	120
2001	111
2002	104
2003	64
2004	40
2005	30

10. Graph the data from the table for the number of sunspots as a function of the year since 1990. Label both axes on the graph with an appropriate scale.



My Notes

MATH TERMS

A function value $f(a)$ is a **relative maximum** of f if there is an interval around a where, for any x in that interval, $f(a) \geq f(x)$.

A function value $f(a)$ is a **relative minimum** of f if there is an interval around a where, for any x in that interval, $f(a) \leq f(x)$.

Relative maxima and minima are often referred to as **turning points**.

My Notes

TECHNOLOGY TIP

Be sure to begin with a viewing window within which all the points of the table will be included. Then expand it to see your function without domain restrictions.

TECHNOLOGY TIP

A graphing calculator can be used to find the turning points of a polynomial function. Use the same method that you would use to find a maximum or a minimum of a quadratic function.

11. What type of function could best be used to model the data? Explain your reasoning.

· CUBIC, THE # OF SUNSPOTS DECREASES, THEN INCREASES, THEN DECREASES AGAIN.

12. Use appropriate tools strategically. Use the regression capabilities of your graphing calculator to find a model to represent the data.

$f(x) = -.605x^3 + 14.937x^2 - 103.359x + 239.81$

13. List the important features of the graph of the function you found in Item 12 without the domain restrictions of the context. In other words, look at the graph of the function defined over the set of real numbers.

REL MIN @ (4.92, 21.3)
REL MAX @ (11.6, 113)
y-INT : (0, 239.81)
ZERO : 15.3

14. Do you think the model from Item 12 would be appropriate for predicting sunspot data 25 years into the future? Explain.

NO, THE # OF SUNSPOTS WOULD BECOME NEGATIVE.

15. Could you use this model to predict the number of sunspots observed by the Chinese in 200 B.C.? Explain.

NO, THE # OF SUNSPOTS WOULD BECOME UNREASONABLY LARGE.

Lesson 9-1
Data and Polynomial Models

Check Your Understanding

Use notebook paper to write your answers.

16. Find a model to represent the data.

x	−4	−3	−1	0	1	4	6
y	−3	0	1	1.5	1	−2	−8

17. Graph the equation you found in Item 16. List the important features of the graph.

18. Describe the characteristics of a data set that could be accurately modeled by a linear function.

LESSON 9-1 PRACTICE

19. Examine the data in the table. What type of function could be used to model the data? Explain your reasoning.

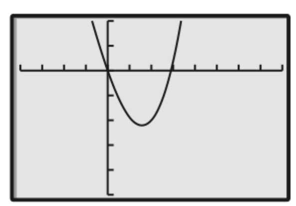
x	1	2	3	4	5	6	7	8	9	10
y	49	22	16	12	10	14	19	17	10	3

20. Use the regression capabilities of your graphing calculator to find a model that best represents the data in Item 19.

21. Attend to precision. Graph the equation you found in Item 19. List the important features of the graph. Approximate any values to three decimal places.

22. An insect population doubles each month. What type of function could be used to model the data for the number of insects as a function of the number of months since January 2012? Explain your reasoning.

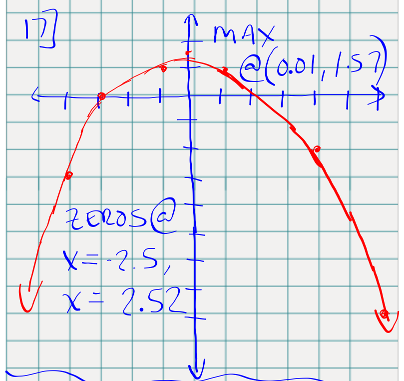
23. Critique the reasoning of others. Jenna says the graph on her calculator, as shown below, must represent a quadratic function. Is she correct? Explain your reasoning.



My Notes

16] $y = -0.2596x^2 + 0.005x + 1.571$

17]



18]. CONSTANT DIFF. IN INPUTS & OUTPUTS
• ALWAYS INCREASING OR ALWAYS DECREASING

My Notes

Learning Targets:

- Describe and analyze graphs of polynomial functions.
- Graph polynomial functions using technology.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Group Presentations, Look for a Pattern, Quickwrite

MATH TERMS

End behavior of a function can be determined by seeing what happens to the graph of a function on the extreme left and right ends of the *x*-axis. See what happens to *y* as *x* approaches $-\infty$ and ∞.

You used a quadratic function and a cubic function to model the sunspot data. These are examples of *polynomial functions.* A ***polynomial function*** of degree *n* is one that can be written in the form

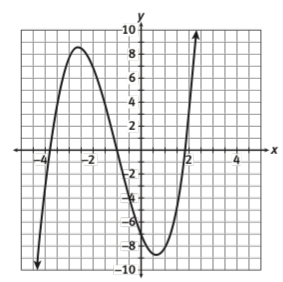
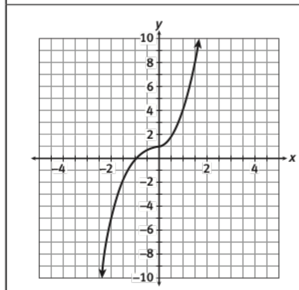
$$f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$$

where n is a nonnegative integer and the coefficients $a_0, a_1, \ldots a_n$ are real numbers with a leading coefficient $a_n \neq 0$.

1. Make sense of problems. Describe the important features of the graphs below.

MATH TIP

Some important features of graphs are the *x*-intercept(s) or zeros of a function, the *y*-intercept, and the turning points, also known as the relative maximum and minimum values of a function.

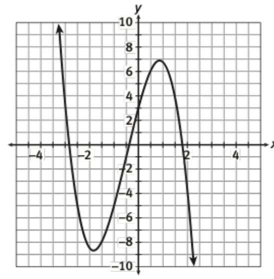
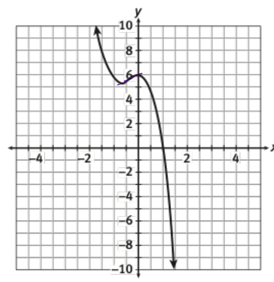
a. $f(x) = x^3 + 3x^2 - 5x - 7$	Features of the Graph
	REL MAX: (-2.63, 8.71) REL MIN: (0.633, -8.71) ZEROS: -3.83, -1, 1.83 Y-INT: (0, -7) END BEHAVIOR: AS $x \to -\infty$, $f(x) \to -\infty$ AS $x \to \infty$, $f(x) \to \infty$ DOMAIN/RANGE: D: $(-\infty, \infty)$ R: $(-\infty, \infty)$ INCREASE/DECREASE: INC: $(-\infty, -2.63) \cup (0.633, \infty)$; DEC: $(-2.63, 0.633)$
b. $f(x) = x^3 + x^2 + x + 1$	Features of the Graph
	REL MAX: NONE REL MIN: ZEROS: -1 Y-INT: (0, 1) E.B. AS $x \to -\infty$, $f(x) \to -\infty$ AS $x \to \infty$, $f(x) \to \infty$ DOMAIN/RANGE: D: $(-\infty, \infty)$ R: $(-\infty, \infty)$ INC: $(-\infty, \infty)$ DEC: NONE

MATH TERMS

A function is said to be **increasing** on intervals of the domain where the graph is rising and **decreasing** on intervals of the domain where the graph is falling.

Lesson 9-2
Polynomial Functions

My Notes

c. $f(x) = -2x^3 - 3x^2 + 8x + 3$	Features of the Graph
	REL MAX: (0.758, 6.47) REL MIN: (−1.76, −9.47) ZEROS: −2.75, −0.341, 1.6 Y-INT (0, 3) E.B. AS $x \to -\infty$, $F(x) \to \infty$ AS $x \to \infty$, $F(x) \to -\infty$ DOMAIN/RANGE: D: $(-\infty, \infty)$ R: $(-\infty, \infty)$ INC: (−1.76, 0.758) DEC: $(-\infty, -1.76) \cup (0.758, \infty)$
d. $f(x) = -4x^3 - x^2 + x + 6$	Features of the Graph
	REL MAX: (0.217, 6.13) REL MIN: (−0.384, 5.7) ZEROS: 1.13 Y-INT: (0, 6) E.B.: AS $x \to -\infty$, $F(x) \to \infty$ AS $x \to \infty$, $F(x) \to -\infty$ DOMAIN/RANGE: D: $(-\infty, \infty)$ R: $(-\infty, \infty)$ INC: (−.384, 0.217) DEC: $(-\infty, -.384) \cup (0.217, \infty)$

2. Compare and contrast the end behavior of cubic functions to the end behavior of quadratic functions.

QUADRATICS REMAIN THE SAME AS $x \to -\infty, \infty$.

CUBICS GO OPPOSITE AS $x \to -\infty, \infty$

DISCUSSION GROUP TIP

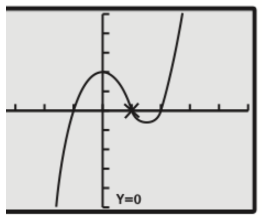
As needed, refer to the Glossary to review translations of key terms. Incorporate your understanding into group discussions to confirm your knowledge and use of key mathematical language.

My Notes

NOLOGY TIP

hing calculator can be used the zeros, the relative um, and the relative um points on a graph.

$- 2x^2 - 9x + 18$



H TERMS

umber of times a given omial function has a factor **multiplicity** of the related

H TERMS

tiple root occurs when a omial equation has a root ultiplicity of 2 or greater. ample, $(x + 3)^2 (x - 3)^2$ has *double* t $x = 3$ and also at $x = -3$.

3. Use appropriate tools strategically. Use a graphing calculator to examine the graphs of the following functions. Determine the following:

- the degree of the polynomial
- the leading coefficient
- the end behavior of the polynomial function
- the maximum number of zeros
- the maximum number of turning points (relative maxima and minima)

a. $f(x) = -\frac{1}{2}x^2 - 4x$

DEG: 2
L.C: $-\frac{1}{2}$
E.B. AS $x \to -\infty$, $f(x) \to -\infty$
AS $x \to \infty$, $f(x) \to -\infty$
MAX ZEROS: 2
MAX T.P(S): 1

b. $f(x) = x(x + 2)^2 (x - 2)^2$

DEG: 5
L.C: 1
E.B. AS $x \to -\infty$, $f(x) \to -\infty$
AS $x \to \infty$, $f(x) \to \infty$
MAX ZEROS: 5
MAX T.P(S): 4

$= x(x^2+4x+4)(x^2-4x+4)$
$(x^3+4x^2+4x)(x^2-4x+4)$
$x^5 \ldots$

c. $f(x) = x(x - 1)^2 (x + 2)^2$

$x(x^2-2x+1)(x^2+4x+4)$
$(x^3-2x^2+x)(x^2+4x+4)$
$x^5 \ldots$

d. $f(x) = -2x^4 + 6x^2 + 4$

e. $f(x) = x^2 + 5x + 7$

f. $f(x) = -2x^5 - 3x^3 + 8x$

g. $f(x) = 4x^5 - 8x^4 - 5x^3 + 10x^2 + x - 1$

h. $f(x) = -\frac{1}{4}x^4 + 3x^2 - 3$

i. $f(x) = x^4 + 2$

STD FORM:

$$x^4 + x^3 + 2x^2 - 5x + 2$$

My Notes

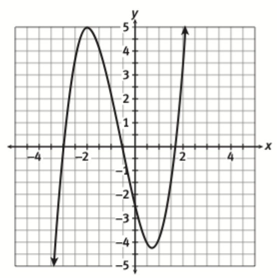
4. For each of the functions in Items 3a to 3d, find the domain and the range of the function, the zeros, the relative maximum and relative minimum points, and the intervals of the domain where the function is increasing and decreasing.

5. What is the relationship between double roots of a polynomial and relative maximum and minimum values?

Check Your Understanding

Use notebook paper to write your answers.

6. Reason abstractly. List the important features of the graph below. Approximate any values to three decimal places.



7. Without using a calculator, list end behavior and x- and y-intercepts of the function.

$f(x) = (x + 2)(x - 2)(x + 5)(x - 5)(x + 7)$

8. Use a graphing calculator to find the zeros, turning points, y-intercepts, and end behavior.

$f(x) = x^5 + 5x^4 - 68x^3 - 340x^2 + 256x + 1280$

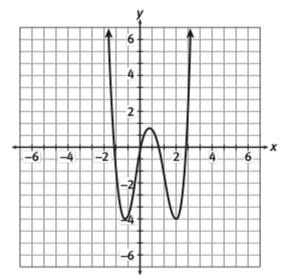
9. Compare and contrast the end behavior of linear functions to the end behavior of quadratic functions.

My Notes

LESSON 9-2 PRACTICE

10. List the degree of each function below.
a. $x^2 + 2x - 4$
b. $x(x - 1)^2$
c. $x(x - 1)^2(x + 1)^2$
d. $(x + 1)(x^2 + 2x - 4)$

11. List the important features of the graph below. Approximate any values to three decimal places.



12. Without using a calculator, determine the end behavior and *x*- and *y*-intercepts of the function.

$$f(x) = (x + 4)(x - 4)(x + 1)(x - 1)$$

13. Without using a calculator, find the end behavior, maximum possible zeros, and maximum possible turning points.

$$f(x) = -2x^8 + 2x^5 - x^4 + 5x^2 - 9$$

14. Use appropriate tools strategically. Use a graphing calculator to find the zeros, turning points, *y*-intercepts, and end behavior.

$$f(x) = x^5 + x^4 - 25x^3 - 25x^2 + 144x + 144$$

15. For the function in Item 13, find the domain and the range of the function and the intervals of the domain where the function is increasing and decreasing.

16. Construct viable arguments. Explain how the important features of polynomial functions help you to identify the type of polynomial function that can be used to model a set of data.

ACTIVITY 9 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 9-1

1. Examine the data in the table. What type of function could be used to model the data? Explain your reasoning.

x	−6	−5	−4	−3	−2	−1	0	1	2	3
y	−5	−1	5	9	10	6	5	−1	−7	−13

2. Use the regression capabilities of your graphing calculator to find a model that best represents the data in Item 1.

3. Graph the equation you found in Item 2. List the important features of the graph. Approximate any values to three decimal places.

4. Use the regression capabilities of your graphing calculator to create a model to represent the data in the table.

x	−1	0	1	2	3	5
y	−3	3	3.5	1	−1.5	5

5. Graph the function you found in Item 4 and list the important features of the graph.

6. The graph of data in a data table falls along a curve. Which type of function is *not* an appropriate model for the data in the table?
A. linear
B. exponential
C. quadratic
D. cubic

7. A small community theater makes slight changes to Saturday night ticket prices each week for 6 weeks. They record the data each week. The table below shows total ticket revenue, in dollars, as a function of ticket price.

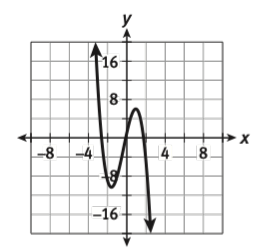
Ticket Price (*x*)	10	15	20	25	30	35
Total Revenue (*y*)	450	645	800	925	780	700

a. Use the regression capabilities of your graphing calculator to find a model that best represents this situation.
b. Use your calculator to graph the equation you found in part a.
c. The theater wants to set ticket prices at the price that will maximize revenue based on the data they recorded. What is the optimal ticket price? Explain your reasoning.

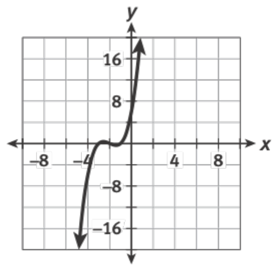
Lesson 9-2

8. List the degree of each function below.
a. $12x^2 + 3x^6 - 14x$
b. $x^3(x - 3)^2$
c. $(x + 7)^2(x - 2)^2$

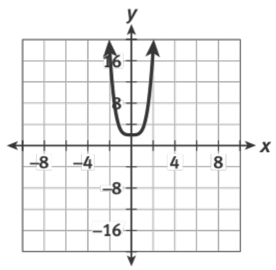
9. List the important features of each graph. Approximate any values to three decimal places.
a. $f(x) = -2x^3 - 2x^2 + 9x + 1$



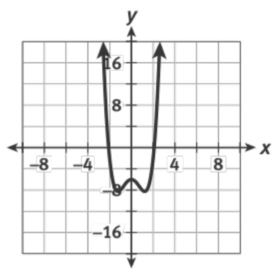
b. $f(x) = x^3 + 6x^2 + 11x + 6$



c. $f(x) = x^4 + 2$



d. $f(x) = x^4 - 3x^2 - 6$



10. Without using a calculator, find the end behavior and x- and y-intercepts of $f(x) = (x + 4)(x - 3)(x + 3)(x - 9)(x + 2)$.

11. Without using a calculator, find the end behavior, maximum possible zeros, and maximum possible turning points of

$f(x) = 5x^9 + 6x^5 - 3x^3 + 5x - 4$.

12. Use a graphing calculator to find the zeros, turning points, y-intercepts, and end behavior of $f(x) = x^4 - 5x^3 - 30x^2 + 40x + 64$.

13. Use a graphing calculator to examine the graphs of the following functions. Determine

- the degree of the polynomial
- the leading coefficient
- the end behavior of the polynomial function
- the maximum number of zeros
- the maximum number of turning points (relative maxima and minima)

a. $f(x) = \frac{1}{2}x^2 + 2x$

b. $f(x) = x(x + 1)^2(x - 1)^2$

c. $f(x) = x^3 + 10x^2 + 31x + 30$

14. For the functions in Item 13, parts a–c, find the domain and the range of the function, zeros, relative maximum and relative minimum points, and the intervals of the domain where the function is increasing and decreasing.

15. Roberto claims that if he knows the degree of a polynomial function, he can correctly determine exactly how many turning points are in the function's graph. Is Roberto correct? If not, explain his error.

16. Write to another student describing a pattern you have recognized among polynomials of different degrees. Write to explain how the pattern is related to features of the various functions.

MATHEMATICAL PRACTICES
Look For and Make Use of Structure

17. Explain how zeros and end behavior of polynomial functions and their graphs are related to the degree and the factors of the polynomial.